Delay-Optimal Streaming Codes under Source-Channel Rate Mismatch

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Overview

This work studies **low delay error correcting codes for streaming** over packet erasure channels. We investigate a practical streaming situation with unequal source frame arrival and channel packet transmission rates that exposes an important design principle for the construction of delay sensitive streaming codes.

Main contributions:

- **Capacity characterization** for the case of burst-erasure channels.
- **Layered approach** for streaming code construction.
- **Robust extension** for protection against i.i.d. isolated erasures.

Motivation

1. Delay is an important issue

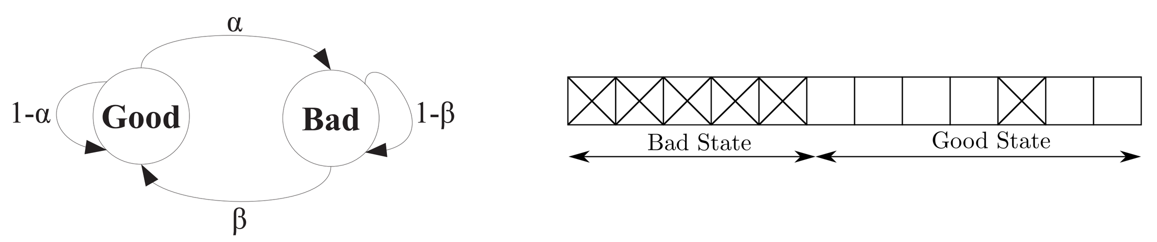
- Strict end-to-end latency requirements for multimedia applications: interactive audio/video conferencing, IPTV, mobile gaming, cloud computing.
- Several sources of delays: processing delay, queuing delay, propagation delay, coding delay.
- We focus on minimizing the coding delay.

2. Why forward error correction (FEC)?

- Feedback not feasible: Multicast streaming scenarios (e.g. digital video broadcasting, IPTV)
- Delayed feedback (possibly lossy): Large round-trip delay preventing ARQ strategies

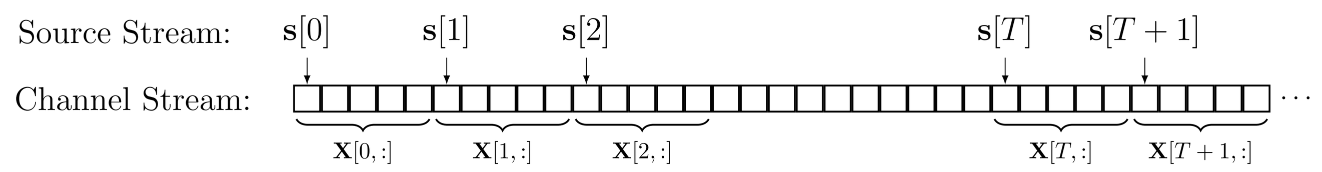
3. Deterministic channel approximation

- Shannon capacity only depends on the fraction of erasures but when delay constraints are imposed, the actual pattern of erasures also become relevant.
- Finding good codes for such channels is a long standing open problem. Our approach is to approximate these models in a deterministic fashion.
- In practice erasures are temporally dependent which is captured by statistical models such as Gilbert channel model.



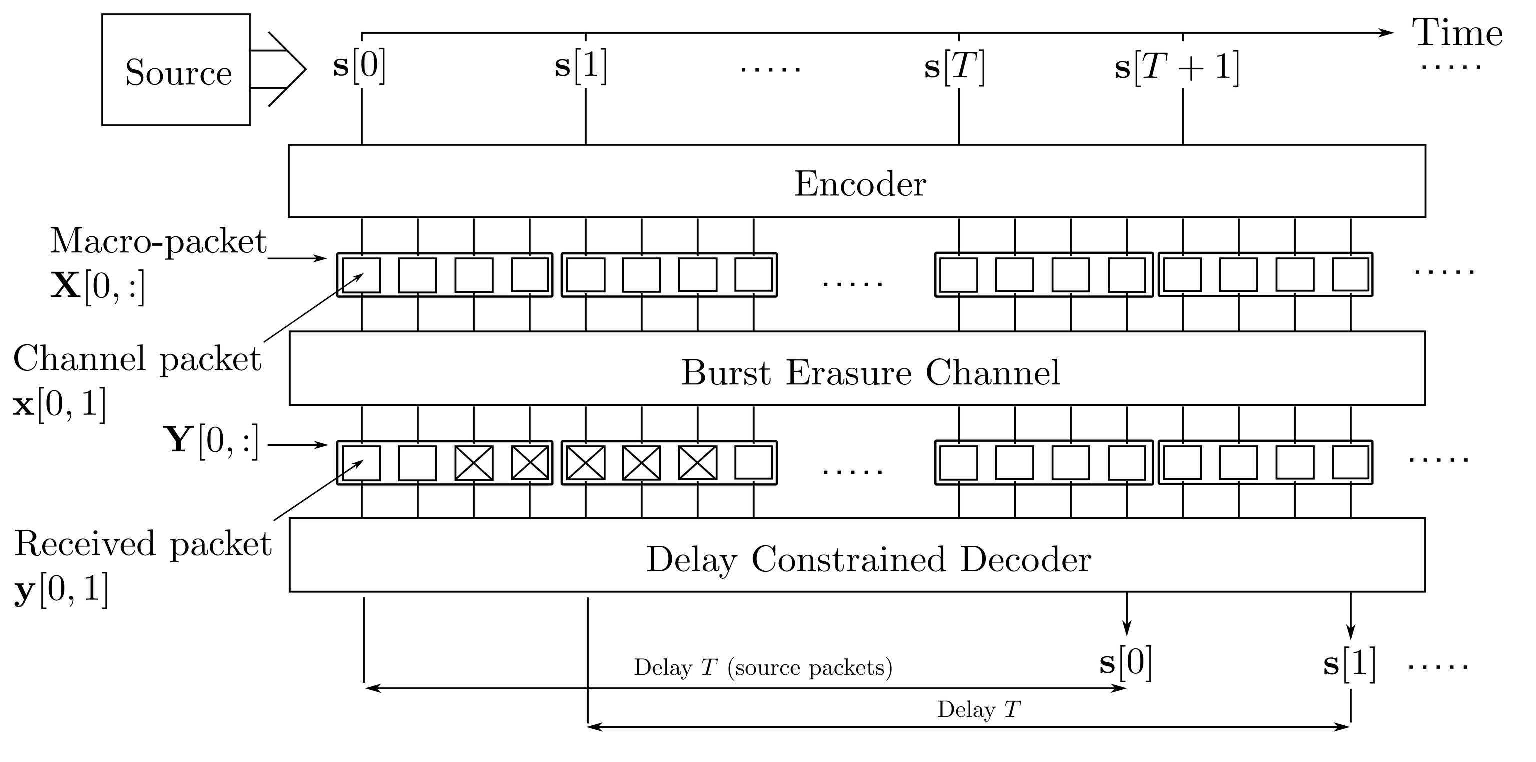
4. Unequal source frame arrival and channel transmission rates

- Large source frames (e.g. video frames - 15 kbytes)
- Channel packet size often limited by the underlying communication protocol (e.g. MTU for ethernet - 1500 bytes)
- Multiple channel packets transmitted between successive source frame arrivals



Problem Setup

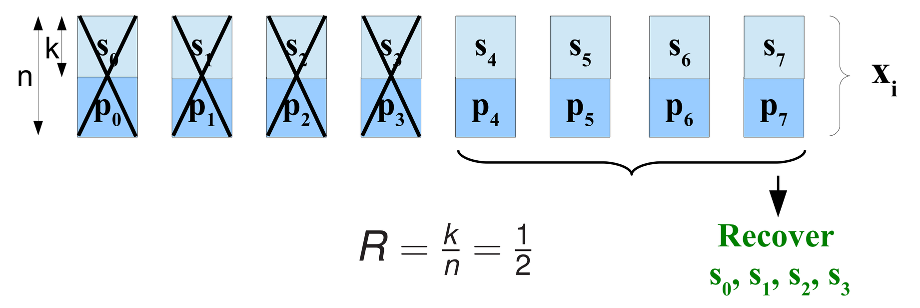
- Streaming source: i.i.d. $\mathbf{s}[i] \sim$ uniform over $\mathbf{F}_q^k$
- Causal encoder: $\mathbf{x}[i,j] = f_{i,j}(\mathbf{s}[0], \mathbf{s}[1], \ldots, \mathbf{s}[i]) \in \mathbf{F}_q^n$, $\mathbf{X}[i,:] = [\mathbf{x}[i,1] \mid \ldots \mid \mathbf{x}[i,M]]$
- Rate of code: $R = \frac{k}{n \times M}$
- Channel: $\mathbf{y}[i,j] = \star$ for burst of maximum B channel packets, otherwise $\mathbf{y}[i,j] = \mathbf{x}[i,j]$
- Delay constrained decoder: $\mathbf{s}[i] = g_i(\mathbf{Y}[0,:], \mathbf{Y}[1,:], \ldots, \mathbf{Y}[i+T,:])$



Questions: $C(M,B,T) =$? Optimal encoding $f_{i,j}$ and decoding g_i functions?

Equal source arrival and channel transmission rates ($M = 1$)

- Traditional coding approach:



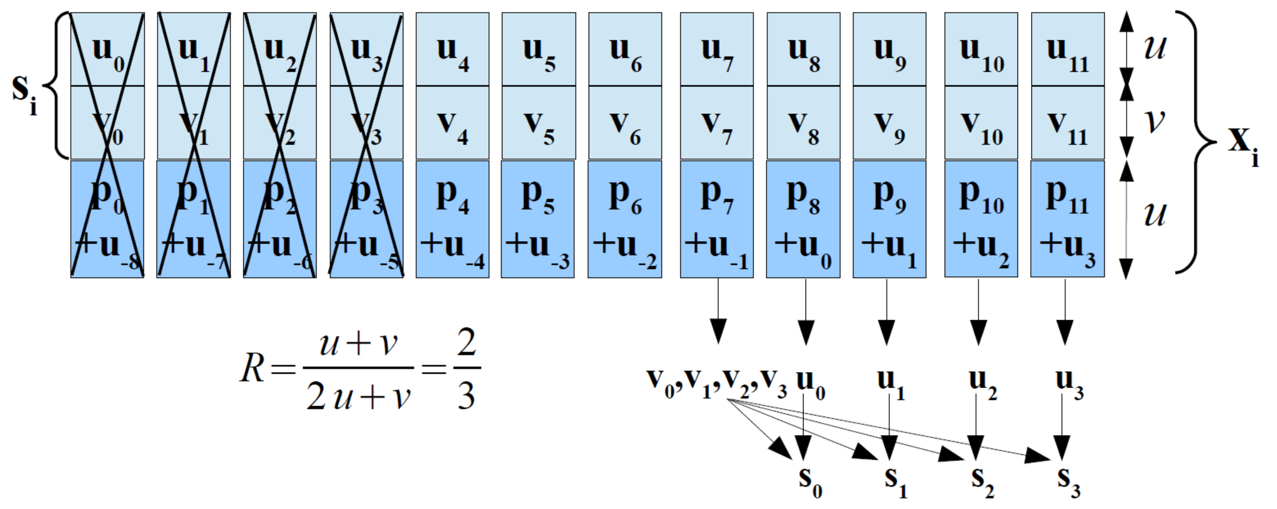
$$\mathbf{p}_i = \mathbf{s}_i \cdot \mathbf{H}_0 + \cdots + \mathbf{s}_{i-M} \cdot \mathbf{H}_M$$

$$\begin{bmatrix} \mathbf{p}_4 \\ \mathbf{p}_5 \\ \mathbf{p}_6 \\ \mathbf{p}_7 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_4 & \mathbf{H}_3 & \mathbf{H}_2 & \mathbf{H}_1 \\ \mathbf{H}_5 & \mathbf{H}_4 & \mathbf{H}_3 & \mathbf{H}_2 \\ 0 & \mathbf{H}_5 & \mathbf{H}_4 & \mathbf{H}_3 \\ 0 & 0 & \mathbf{H}_5 & \mathbf{H}_4 \end{bmatrix}}_{\text{full rank}} \begin{bmatrix} \mathbf{s}_0 \\ \mathbf{s}_1 \\ \mathbf{s}_2 \\ \mathbf{s}_3 \end{bmatrix}$$

$R = \frac{k}{n} = \frac{1}{2}$

Strongly-MDS codes: Deterministic codes with maximum distance properties

Layered coding approach:



$R = \frac{u+v}{2u+v} = \frac{2}{3}$

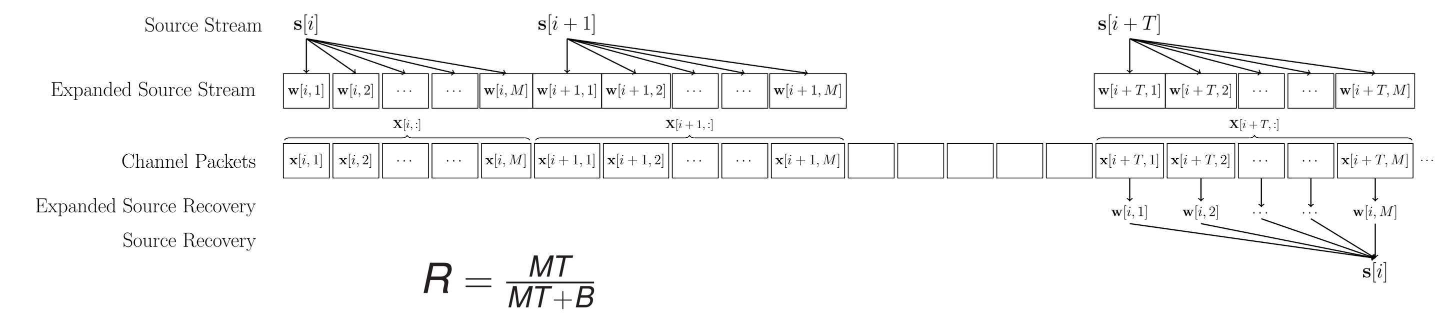
- Main encoding steps:
  1. Source splitting into two groups $\mathbf{u}$ and $\mathbf{v}$
  2. Strongly-MDS code on $\mathbf{v}$ (Parity layer 1)
  3. Repetition code on $\mathbf{u}$ (Parity layer 2)
  4. Final parity checks = Parity layer 1 + Parity layer 2

Optimal splitting ratio: $u = B$, $v = T - B$ results into capacity $C = \frac{T}{T+B}$

Unequal source arrival and channel tranmission rates (general M)

- Strongly-MDS code: $R = 1 - \frac{B}{M(T+1)}$
- Straightforward adaptation of layered code for $M = 1$:

First split each source packet into M sub-packets. Then apply layered code for $M = 1$.

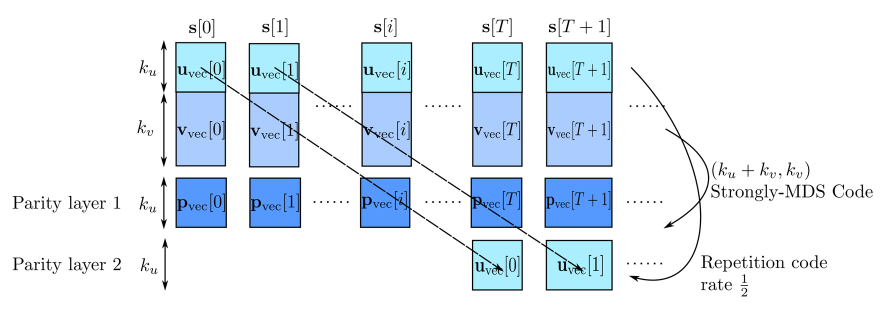


$R = \frac{MT}{MT+B}$

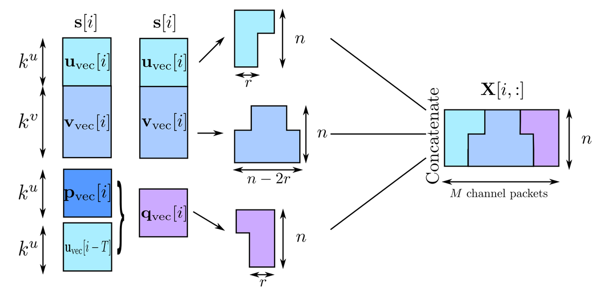
Reshaping approach:

- Encoding:

1. Encode entire macro-packet

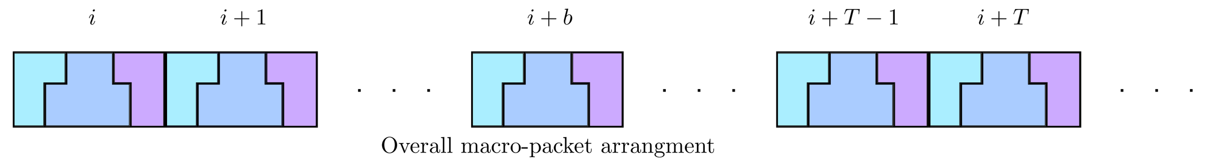


2. Reshape into M channel packets



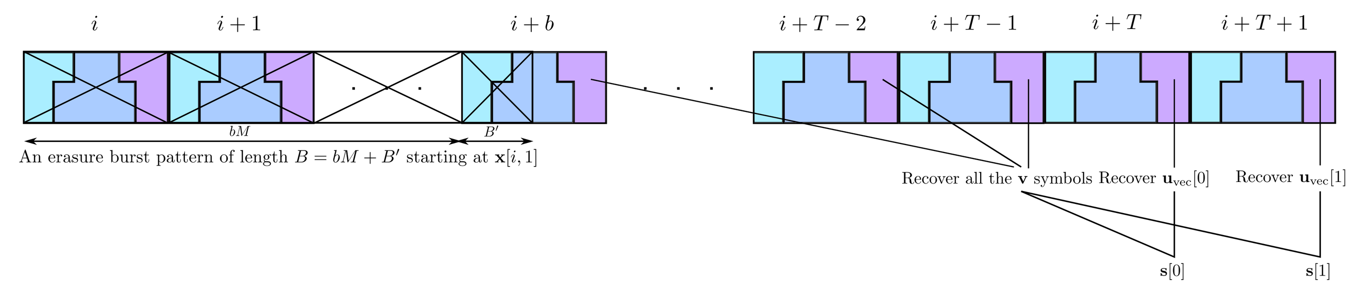
$R = \frac{k_u + k_v}{2k_u + k_v}$

Overall macro-packet structure:



- Decoding analysis:

For the reshaped code, the worst case burst pattern starts at the beginning of the macro-packet.



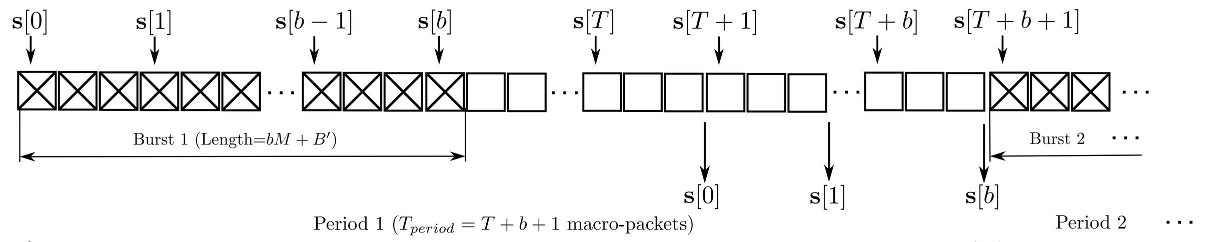
Let $B = bM + B'$. Two cases depending on whether $B' \lessgtr (1-R)M$:

1. Burst only erases symbols from $\mathbf{u}_{\text{vec}}[i+b]$
   - Recovery of $\mathbf{v}$ symbols: $(k_u + k_v)b = k_u T$
   - Optimal spitting ratio: $k_u = b$, $k_v = T - b$
   - $R = \frac{T}{T+b}$

2. Burst erases symbols from $\mathbf{v}_{\text{vec}}[i+b]$
   - Recovery of $\mathbf{v}$ symbols: $(k_u + k_v)b + (B'n - k_u) = k_u T$
   - Optimal spitting ratio: $k_u = B$, $k_v = M(T+b+1) - 2B$
   - $R = \frac{M(T+b+1)-B}{M(T+b+1)}$

Converse

- $T > b$: Technique of periodic erasure channel



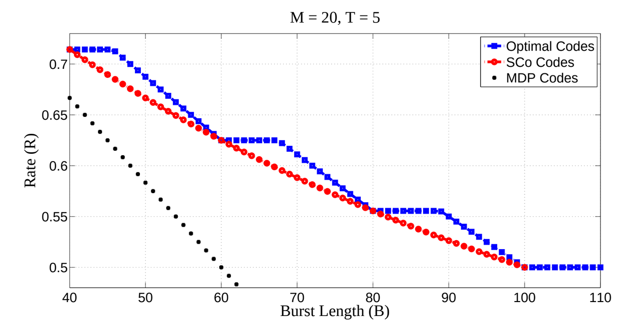
- Enough parities to recover from full burst $bM + B' \implies 1 - R \geq \frac{bM + B'}{M(T+b+1)}$
- Enough parities to recover from burst length $bM \implies 1 - R \geq \frac{bM}{M(T+b)}$
- $T = b$: Periodic channel argument not tight, different argument

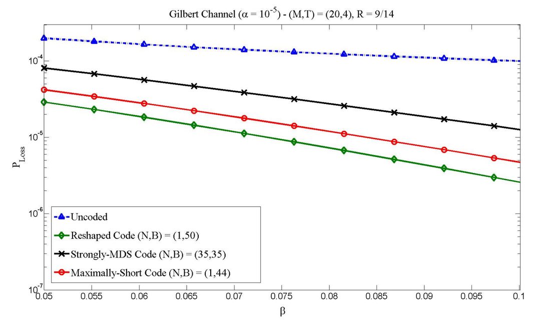
Main capacity result

Theorem: For the given streaming setup, with any M, T and B, the *streaming capacity* C is expressed as follows:

$$C = \begin{cases} \frac{T}{T+b}, & B' \leq \frac{b}{T+b}M,\ T \geq b, \\ \frac{M(T+b+1)-B}{M(T+b+1)}, & B' > \frac{b}{T+b}M,\ T > b, \\ \frac{M-B'}{M}, & B' > \frac{M}{2},\ T = b, \\ 0, & T < b. \end{cases}$$

Numerical comparision, performance over practical channel



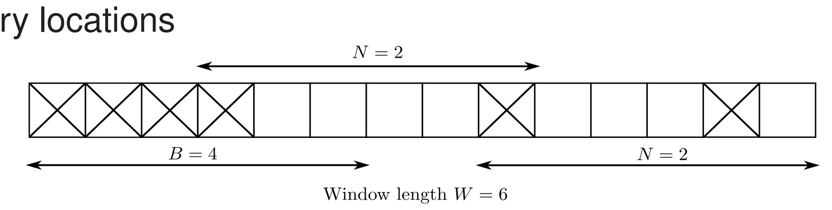


Interesting capacity behavior: Capacity doesn't decrease as burst length goes from cM to $cM + B'$ for $c \in \mathbb{N}$

Robust Extension

- Extending the channel model to account for i.i.d. isolated erasures:

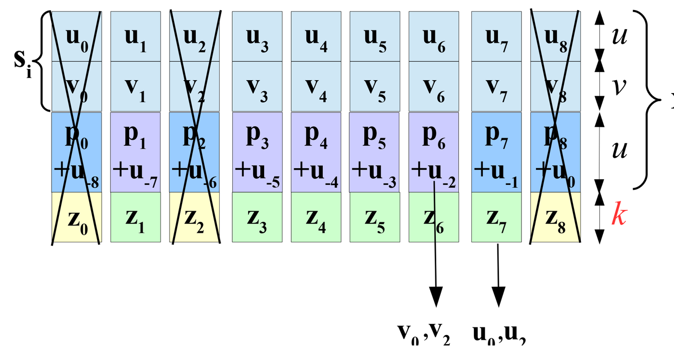
Locally constrained erasure patterns - any sliding window W of length $M(T+1)$ channel packets has either, (i) a single erasure burst of maximum length B, or (ii) a maximum of N erasures in arbitrary locations



Questions: $C(M,N,B,T) =$? Optimal encoding and decoding?

$M = 1$

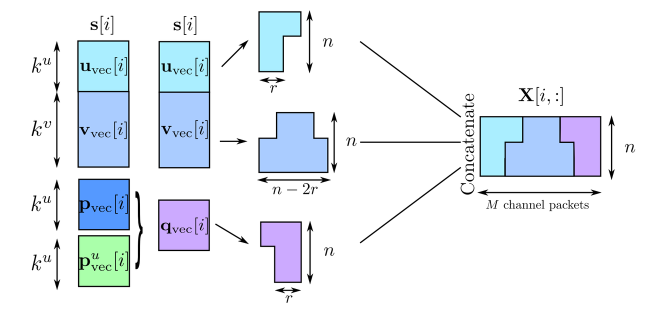
- Append an additional layer parity checks containing Strongly-MDS code on $\mathbf{u}$



- $R = \frac{u+v}{2u+v+k}$
- Very close to being optimal, $k = \frac{N}{T-N+1}B$

General M (achievable approach I):

- Repetition code replaced by Strongly-MDS code for $\mathbf{u}$
- Rate of the code unchanged.
- $N = r$ is achievable.



- Many possibilities for the placement of Strongly-MDS parities for u!

Related Work

- E. Martinian and C. W. Sundberg, "Burst Erasure Correction Codes With Low Decoding Delay," *IEEE Transactions on Information Theory*, October 2004
- A. Badr, A. Khisti, W. Tan and J. Apostoloupolos, "Robust Streaming Erasure Codes based on Deterministic Channel Approximations," *ISIT*, Istanbul, Turkey, July 2013.
- A. Badr, P. Patil, A. Khisti, W. Tan and J. Apostolopoulos "Layered Construction for Low-Delay Streaming Codes" Submitted, *IEEE Trans. Inf. Theory*, August 2013
- D. Leong and T. Ho, "Erasure coding for real-time streaming," in *ISIT*, 2012.
- H. Gluesing-Luerssen, J. Rosenthal, and R. Smarandache, "Strongly MDS convolutional codes," *IEEE Transactions on Information Theory*, 2006.