Layered Constructions for Low-Delay Streaming Codes

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Abstract

- Deterministic Erasure Channel - Bursts or Isolated Erasures
- Fundamental Tradeoff Between Burst and Isolated Erasure Correction Capability
- Low-Delay Streaming Codes with Layered Architecture
- Unequal Source-Channel Frame Rates
- Optimal Code Construction with Reshaping

Streaming Setup - Matched Case

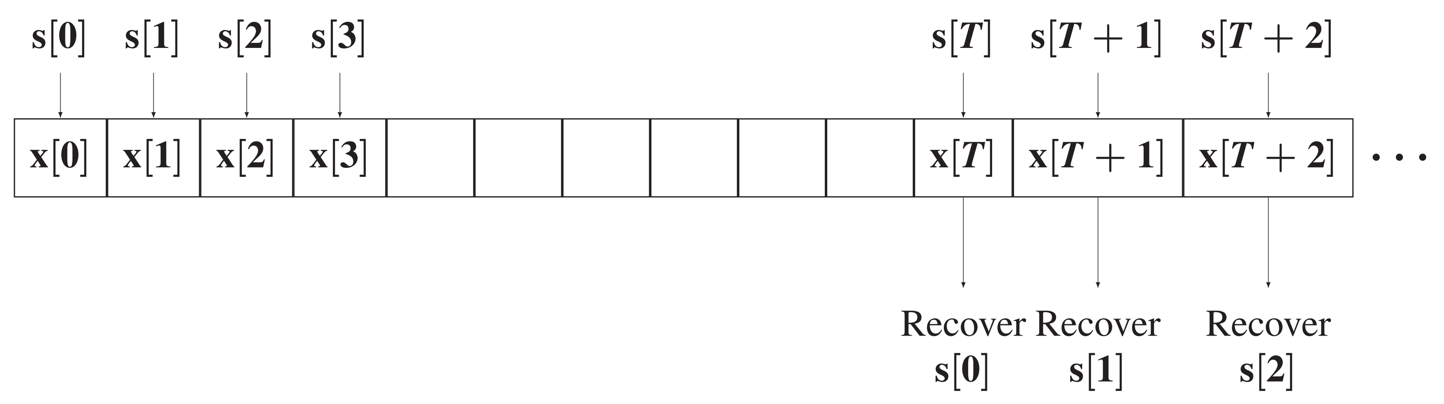


Figure : At each time instant i, a source packet $s[i]$ arrives at the encoder which generates $\mathbf{x}[i]$. The decoder must reconstruct each source packet within a delay of T packets.

- Source Model: $\mathbf{s}[i] \sim \mathbf{Unif}\{\mathbb{F}_q^k\}$ (i.i.d.)
- Streaming Encoder: $\mathbf{x}[i] = f_i(\mathbf{s}[0], \dots, \mathbf{s}[i]) \in \mathbb{F}_q^n$
- Deterministic Erasure Channel: Introduces either,
  - Burst of max. length B or,
  - Up to N isolated erasures
- Delay-Constrained Decoder: $\hat{\mathbf{s}}[i] = g_t(\mathbf{Y}[0,:], \dots, \mathbf{Y}[i+T,:])$
- Rate: $R = \frac{k}{n}$

Baseline Codes

- **Strongly-MDS Codes**

$$R = \frac{T-B+1}{T+1} = \frac{2}{5}$$

	0	1	2	3	4	5	6	7	8
k=2	$s_0[0]$ $s_1[0]$	$s_0[1]$ $s_1[1]$	$s_0[2]$ $s_1[2]$	$s_0[3]$ $s_1[3]$	$s_0[4]$ $s_1[4]$	$s_0[5]$ $s_1[5]$	$s_0[6]$ $s_1[6]$	$s_0[7]$ $s_1[7]$	$s_0[8]$ $s_1[8]$
n-k=3	$p_0[0]$ $p_1[0]$ $p_1[0]$	$p_0[1]$ $p_1[1]$ $p_1[1]$	$p_0[2]$ $p_1[2]$ $p_1[2]$	$p_0[3]$ $p_1[3]$ $p_1[3]$	$p_0[4]$ $p_1[4]$ $p_1[4]$	$p_0[5]$ $p_1[5]$ $p_1[5]$	$p_0[6]$ $p_1[6]$ $p_1[6]$	$p_0[7]$ $p_1[7]$ $p_1[7]$	$p_0[8]$ $p_1[8]$ $p_1[8]$

⇓
Recover
$\mathbf{s}[0], \mathbf{s}[1], \mathbf{s}[2]$

- **Maximally-Short (MS) Codes:** $\mathcal{C}(N=1, B, W)$

$$B \leq T \cdot \min\left(1, \frac{1-R}{R}\right)$$

$$R = \frac{T}{T+B} = \frac{4}{7}$$

	0	1	2	3	4	5	6	7
B = 3	$u_0[0]$ $u_1[0]$ $u_2[0]$	$u_0[1]$ $u_1[1]$ $u_2[1]$	$u_0[2]$ $u_1[2]$ $u_2[2]$	$u_0[3]$ $u_1[3]$ $u_2[3]$	$u_0[4]$ $u_1[4]$ $u_2[4]$	$u_0[5]$ $u_1[5]$ $u_2[5]$	$u_0[6]$ $u_1[6]$ $u_2[6]$	$u_0[7]$ $u_1[7]$ $u_2[7]$
T-B = 1	$v_0[0]$	$v_0[1]$	$v_0[2]$	$v_0[3]$	$v_0[4]$	$v_0[5]$	$v_0[6]$	$v_0[7]$
B = 3	$u_0[-4]$ $+p_0[0]$ $u_1[-4]$ $+p_1[0]$ $u_2[-4]$ $+p_2[0]$	$u_0[-3]$ $+p_0[1]$ $u_1[-3]$ $+p_1[1]$ $u_2[-3]$ $+p_2[1]$	$u_0[-2]$ $+p_0[2]$ $u_1[-2]$ $+p_1[2]$ $u_2[-2]$ $+p_2[2]$	$u_0[-1]$ $+p_0[3]$ $u_1[-1]$ $+p_1[3]$ $u_2[-1]$ $+p_2[3]$	~~$u_0[0]$~~ $+p_0[4]$ ~~$u_1[0]$~~ $+p_1[4]$ ~~$u_2[0]$~~ $+p_2[4]$	~~$u_0[1]$~~ $+p_0[5]$ ~~$u_1[1]$~~ $+p_1[5]$ ~~$u_2[1]$~~ $+p_2[5]$	~~$u_0[2]$~~ $+p_0[6]$ ~~$u_1[2]$~~ $+p_1[6]$ ~~$u_2[2]$~~ $+p_2[6]$	$u_0[3]$ $+p_0[7]$ $u_1[3]$ $+p_1[7]$ $u_2[3]$ $+p_2[7]$

MiDAS Codes: $\mathcal{C}(N > 1, B, W)$

$$T \leq \left(\frac{R}{1-R}\right) B + N \leq T + 1,$$

$$K = \frac{NB}{T-N+1} = 2$$

$$R = \frac{T}{T+B+K} = \frac{4}{9}$$

	0	1	2	3	4	5	6	7
B = 3	$u_0[0]$ $u_1[0]$ $u_2[0]$	$u_0[1]$ $u_1[1]$ $u_2[1]$	$u_0[2]$ $u_1[2]$ $u_2[2]$	$u_0[3]$ $u_1[3]$ $u_2[3]$	$u_0[4]$ $u_1[4]$ $u_2[4]$	$u_0[5]$ $u_1[5]$ $u_2[5]$	$u_0[6]$ $u_1[6]$ $u_2[6]$	$u_0[7]$ $u_1[7]$ $u_2[7]$
T-B = 1	$v_0[0]$	$v_0[1]$	$v_0[2]$	$v_0[3]$	$v_0[4]$	$v_0[5]$	$v_0[6]$	$v_0[7]$
B = 3	$u_0[-4]$ $+p_0[0]$ $u_1[-4]$ $+p_1[0]$ $u_2[-4]$ $+p_2[0]$	$u_0[-3]$ $+p_0[1]$ $u_1[-3]$ $+p_1[1]$ $u_2[-3]$ $+p_2[1]$	$u_0[-2]$ $+p_0[2]$ $u_1[-2]$ $+p_1[2]$ $u_2[-2]$ $+p_2[2]$	$u_0[-1]$ $+p_0[3]$ $u_1[-1]$ $+p_1[3]$ $u_2[-1]$ $+p_2[3]$	$u_0[0]$ $+p_0[4]$ $u_1[0]$ $+p_1[4]$ $u_2[0]$ $+p_2[4]$	$u_0[1]$ $+p_0[5]$ $u_1[1]$ $+p_1[5]$ $u_2[1]$ $+p_2[5]$	$u_0[2]$ $+p_0[6]$ $u_1[2]$ $+p_1[6]$ $u_2[2]$ $+p_2[6]$	$u_0[3]$ $+p_0[7]$ $u_1[3]$ $+p_1[7]$ $u_2[3]$ $+p_2[7]$
K = 2	$q_0[0]$ $q_1[0]$	$q_0[1]$ $q_1[1]$	$q_0[2]$ $q_1[2]$	$q_0[3]$ $q_1[3]$	$q_0[4]$ $q_1[4]$	$q_0[5]$ $q_1[5]$	$q_0[6]$ $q_1[6]$	$q_0[7]$ $q_1[7]$

Column-Distance Column-Span Tradeoff (d_T, c_T)

$$\mathbf{x}_{[0:T]} = \left[\mathbf{x}[0], \mathbf{x}[1], \dots, \mathbf{x}[T]\right] = \left[\mathbf{s}[0], \mathbf{s}[1], \dots, \mathbf{s}[T]\right] \cdot \mathbf{G}_T^s.$$

Column-Distance: $d_T = \min_{\mathbf{x} \in \mathcal{C}}\{wt(\mathbf{x}_{[0:T]}) | \mathbf{s}[0] \neq \mathbf{0}\} = N + 1$

Column-Span: $c_T = \min_{\mathbf{x} \in \mathcal{C}}\{span(\mathbf{x}_{[0:T]}) | \mathbf{s}[0] \neq \mathbf{0}\} = B + 1$

Tradeoff: $T + \frac{1}{1-R} \leq \left(\frac{R}{1-R}\right) c_T + d_T \leq T + 1 + \frac{1}{1-R},$

Streaming Setup - Mismatched Case

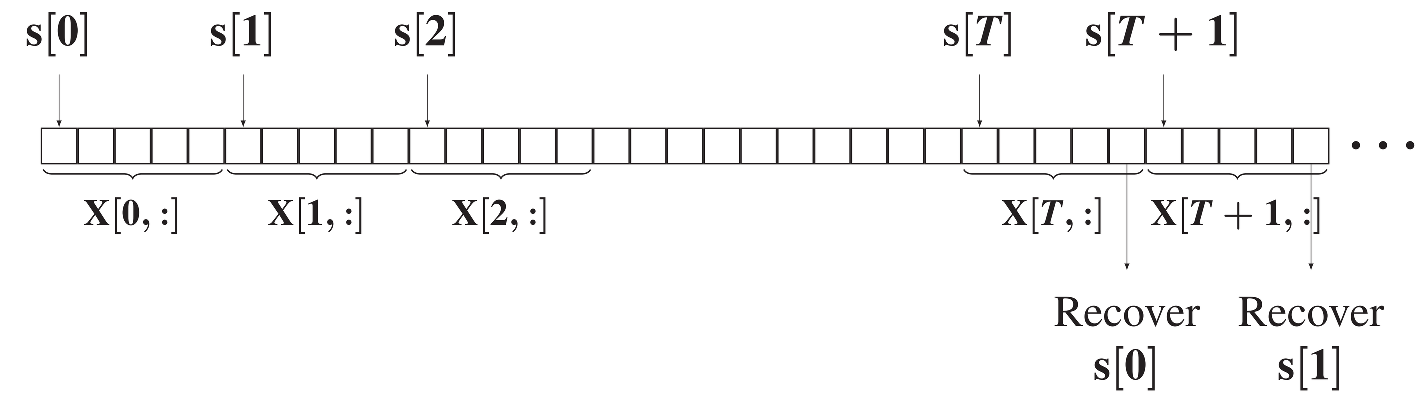
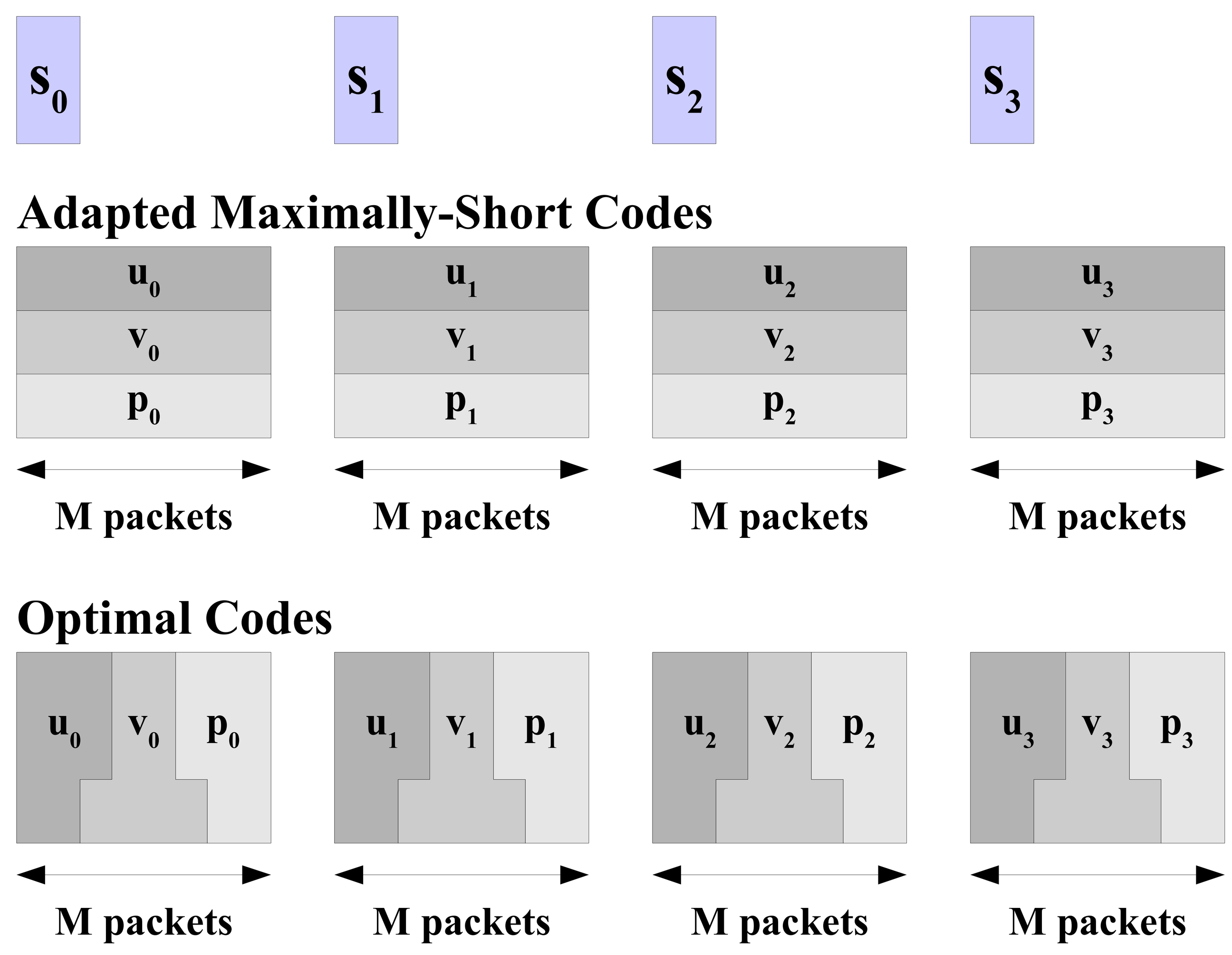


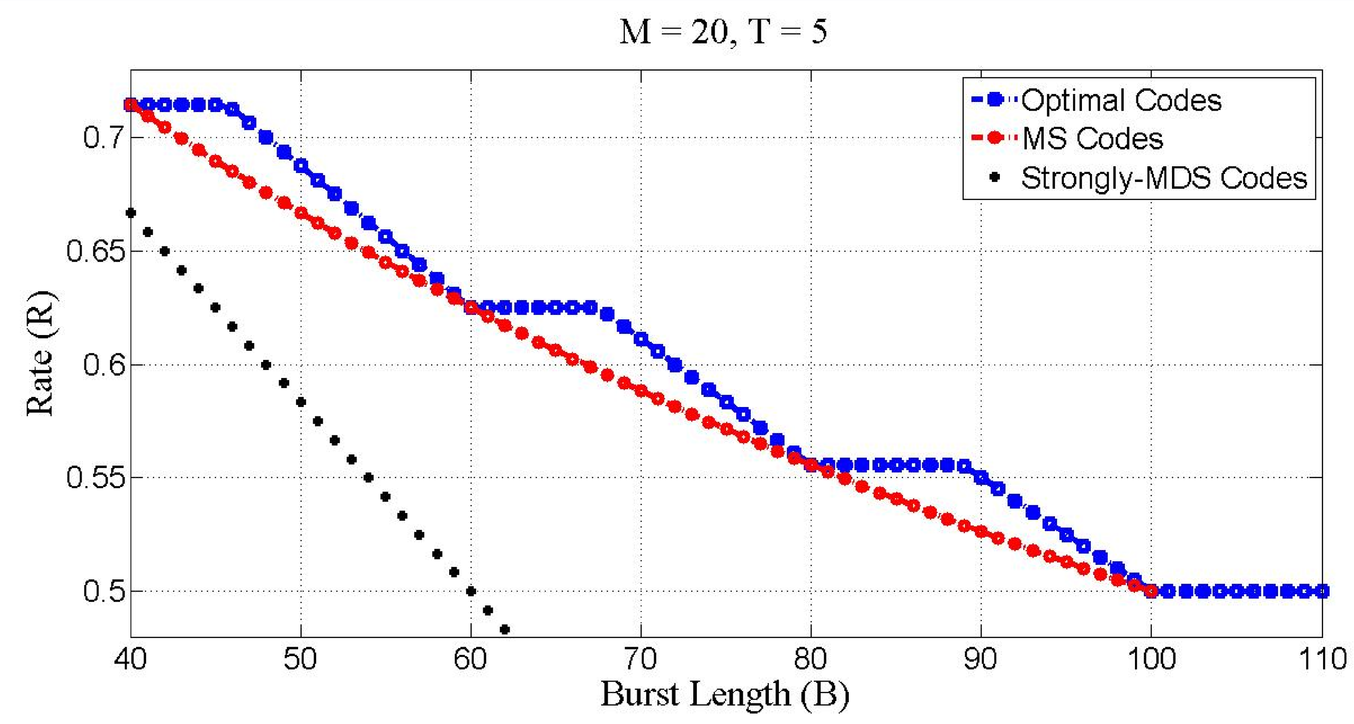
Figure : Each source packet $\mathbf{s}[i]$ arrives just before the transmission of $\mathbf{X}[i,:]$ and needs to be reconstructed by the destination after a delay of T macro-packets.

- Streaming Encoder: $\mathbf{X}[i,:] = [\mathbf{x}[i,1]| \dots |\mathbf{x}[i,M]]$.
- Burst Erasure Channel: $\mathcal{C}(N=1, B, W \geq M(T+1))$
- Delay-Constrained Decoder: $\hat{\mathbf{s}}[i] = g_t(\mathbf{Y}[i,:], \dots, \mathbf{Y}[i+T,:])$
- Rate: $R = \frac{k}{Mn}$

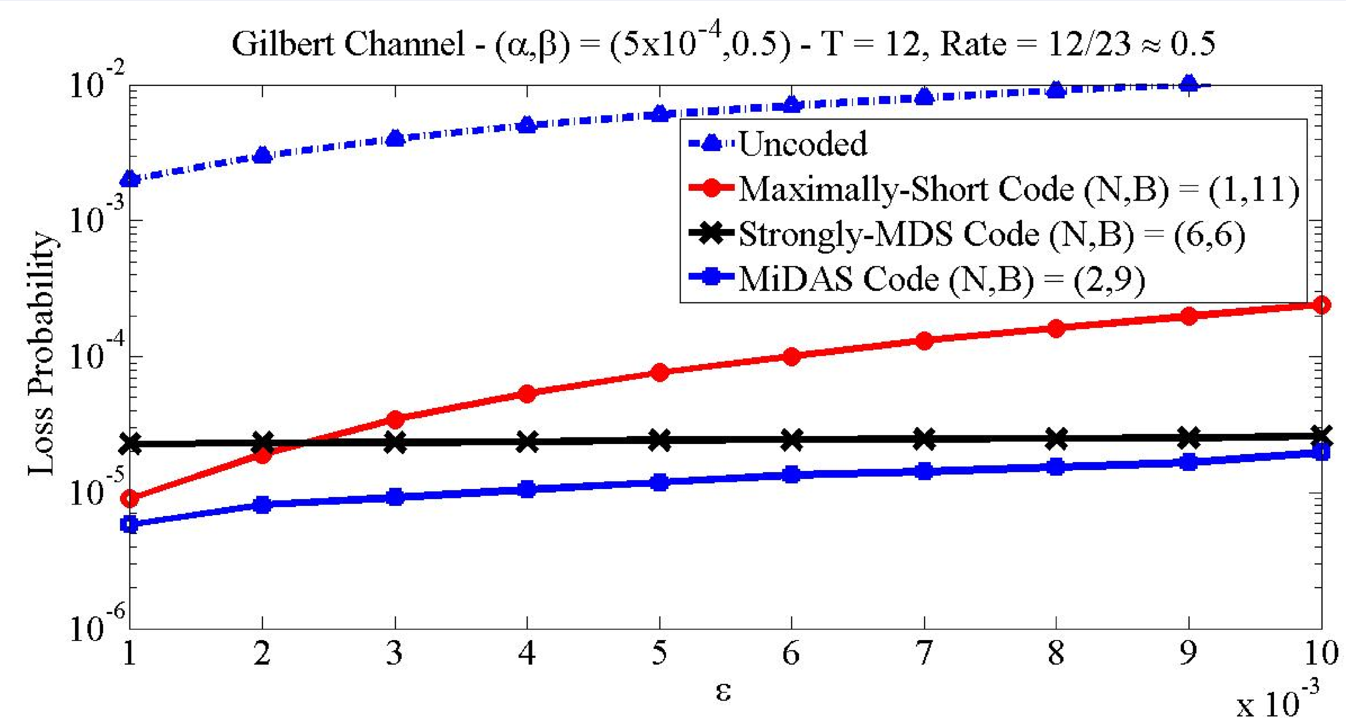
Code Construction



Numerical Comparison



Simulation Results



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