# Delay-Optimal Streaming Codes under Source-Channel Rate Mismatch

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# Motivation - Delay Sensitive Communication

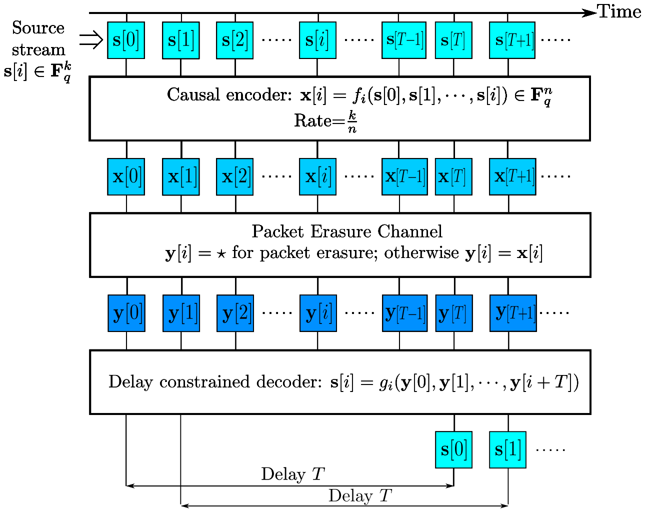
- Delay is a central issue in many applications[1]

| Application | Bit-Rate | MSDU (B) | Delay (ms) | Delay (pkts) | PLR |
|---|---|---|---|---|---|
| Video Conf. | 2 Mbps | 1500 | 100 ms | 16 | $10^{-4}$ |
| Interactive Gaming | 1Mbps | 512 | 50 ms | 12 | $10^{-4}$ |
| SDTV | 4Mbps | 1500 | 200 ms | 66 | $10^{-6}$ |

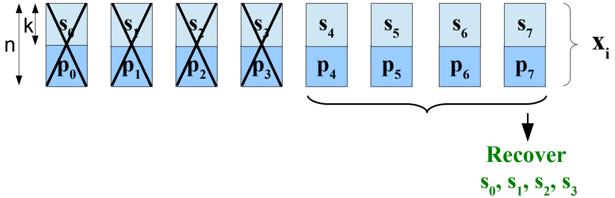
- Various delays: network delay - processing delay, propagation delay, queuing delay and coding delay.
- Communication medium: wireless channel.
- Why forward error correction (FEC) and not automatic repeat request (ARQ)?
  - Multicast streaming: feedback not feasible (e.g. digital video broadcasting, IPTV)
  - Large round-trip delay: delayed feedback (possibly lossy too)

[1] IEEE Usage Model Proposal (doc.: IEEE 802.11-03/802r23)

# Real-time Streaming Setup



# Error Correction for Burst Erasure Channel: Baseline Techniques




$$\mathbf{p}_i = \mathbf{s}_i \cdot \mathbf{H}_0 + \mathbf{s}_{i-1} \cdot \mathbf{H}_1 + \ldots + \mathbf{s}_{i-M} \cdot \mathbf{H}_M, \qquad \mathbf{H}_i \in \mathbb{F}_q^{k \times n-k}$$

Erasure Codes:

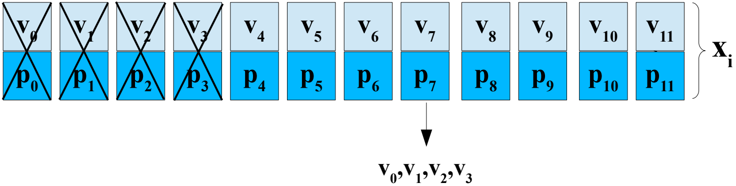
- Random Linear Codes
- Strongly-MDS Codes (Gabidulin'88, Gluesing-Luerssen'06)

$$\begin{bmatrix} \mathbf{p}_4 \\ \mathbf{p}_5 \\ \mathbf{p}_6 \\ \mathbf{p}_7 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_4 & \mathbf{H}_3 & \mathbf{H}_2 & \mathbf{H}_1 \\ \mathbf{H}_5 & \mathbf{H}_4 & \mathbf{H}_3 & \mathbf{H}_2 \\ 0 & \mathbf{H}_5 & \mathbf{H}_4 & \mathbf{H}_3 \\ 0 & 0 & \mathbf{H}_5 & \mathbf{H}_4 \end{bmatrix}}_{\text{full rank}} \begin{bmatrix} \mathbf{s}_0 \\ \mathbf{s}_1 \\ \mathbf{s}_2 \\ \mathbf{s}_3 \end{bmatrix}$$

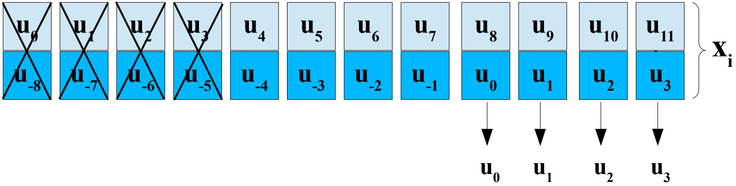
# Streaming Codes - Burst Erasure Channel

$B = 4,\ T = 8$

Rate 1/2 Baseline Erasure Codes, $T = 7$

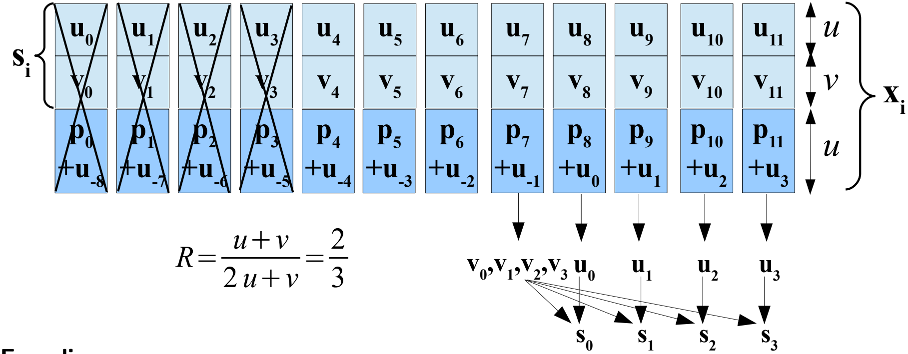



Rate 1/2 Repetition Code, $T = 8$



# Burst-Erasure Streaming Codes

$B = 4,\ T = 8$



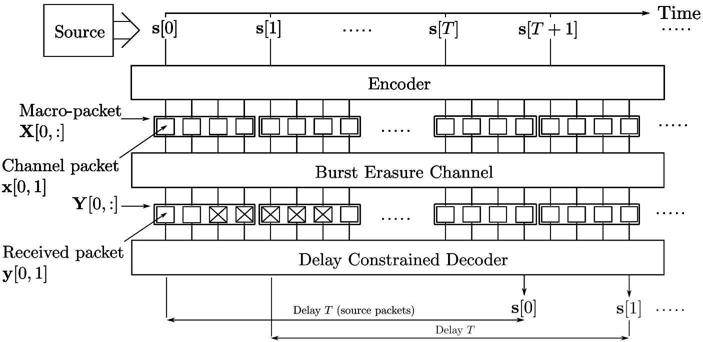

$$R = \frac{u+v}{2u+v} = \frac{2}{3}$$

**Encoding:**

1. Split each source symbol into 2 groups $\mathbf{s}_i = (\mathbf{u}_i, \mathbf{v}_i)$
2. Apply Erasure code to the $\mathbf{v}_i$ stream generating $\mathbf{p}_i$ parities
3. Repeat the $\mathbf{u}_i$ symbols with a shift of $T$
4. Combine the repeated $\mathbf{u}_i$'s with the $\mathbf{p}_i$'s

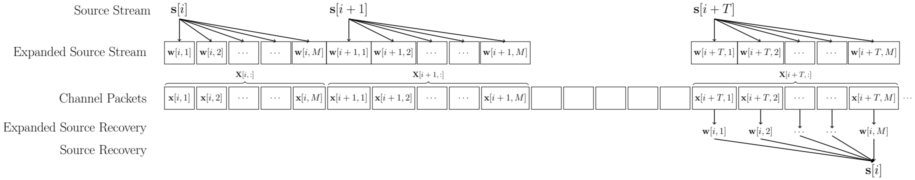
- Choosing $u = B$ and $v = T - B$, $R = \frac{T}{T+B}$ (Optimal) [Badr, Khisti-Infocom '13]
- Capacity first analyzed by Martinian and Sundberg (IT-2004) (Maximally Short Codes)

# Streaming Setup under Source-Channel Rate Mismatch



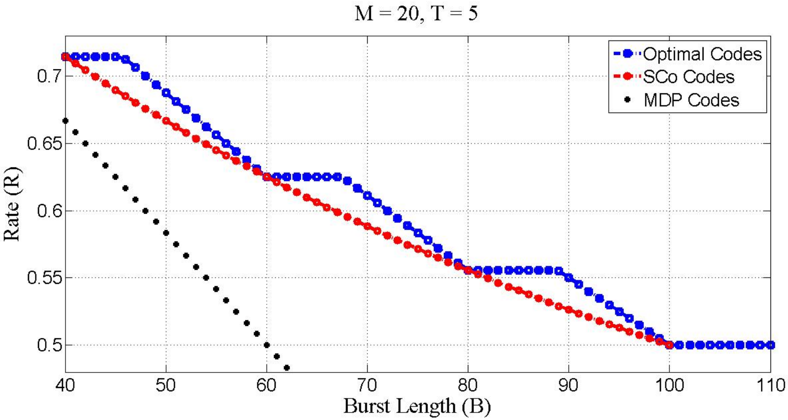

- Source model: i.i.d. process with $\mathbf{s}[i] \sim$ uniform over $\mathbf{F}_q^k$
- Streaming encoder: $\mathbf{x}[i,j] = f_{i,j}(\mathbf{s}[0], \mathbf{s}[1], \cdots, \mathbf{s}[i]) \in \mathbf{F}_q^n$
  **Macro-packet**: $\mathbf{X}[i,:] = [\mathbf{x}[i,1] \mid \ \ldots \ \mid \mathbf{x}[i,M]]$
  **Rate**: $R = \frac{\mathrm{H}(\mathbf{s})}{n \times M} = \frac{k}{n \times M}$
- Packet erasure channel: erasure burst of maximum $B$ channel packets
- Delay-constrained decoder: $\mathbf{s}[i]$ needs to be recovered by macro-packet $i+T$

# Adaptation of Code Construction for M=1 to any M




- Split each source-packet $\mathbf{s}[i]$ into $M$ sub-source-packets $\mathbf{w}[i,1], \cdots, \mathbf{w}[i,M]$
- Apply code for $M = 1$ to this expanded source stream
- Modified delay (w.r.t.) each sub-source-packet is $MT$, burst length is still $B$
- Achievable rate: $R = \frac{MT}{MT+B}$
- Only optimal for $B = cM$, for any $c \in \mathbb{N}$

# Numerical Comparison



# Optimal Code Construction

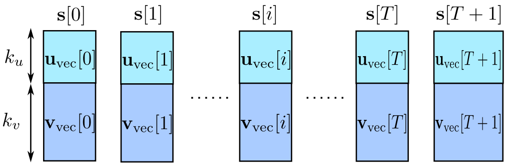
Key Idea:

1. Apply code for $M = 1$ on complete source-packet to generate the entire macro-packet.
2. Map/reshape the generated macro-packet into $M$ individual channel-packets.

# Optimal Code Construction (cont.)

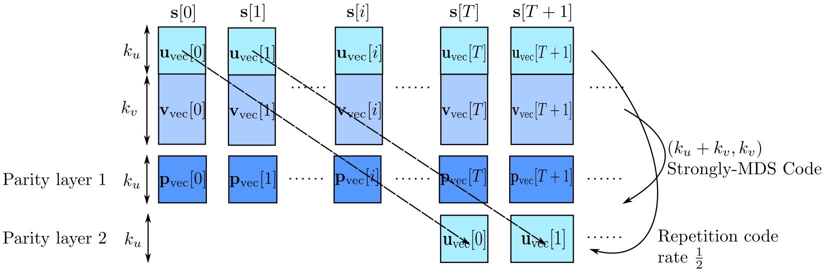
1. Source splitting
   - split $\mathbf{s}[i]$ into into two groups

$$
\begin{aligned}
\mathbf{s}[i] &= (s_1[i], \ldots, s_k[i]) \\
&= (\underbrace{u_1[i], \ldots, u_{k_u}[i]}_{\mathbf{u}_{\mathrm{vec}}[i]}, \underbrace{v_1[i], \ldots, v_{k_v}[i]}_{\mathbf{v}_{\mathrm{vec}}[i]})
\end{aligned}
$$



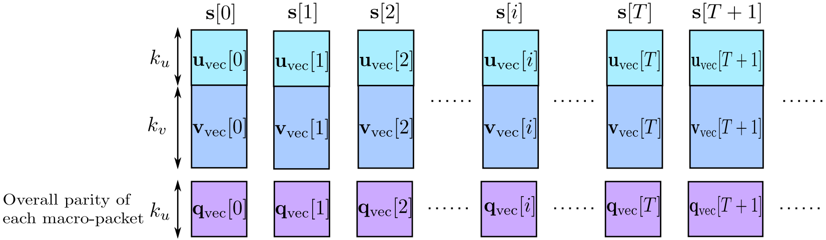
# Optimal Code Construction (cont.)

2. Parity generation
   - layer 1: $(k_v + k_u, k_v, T)$ Strongly-MDS code applied to $\mathbf{v}_{\text{vec}}[\cdot]$ generating $\mathbf{p}_{\text{vec}}[i]$
   - layer 2: repetition code on $\mathbf{u}_{\text{vec}}$ with a shift of $T$



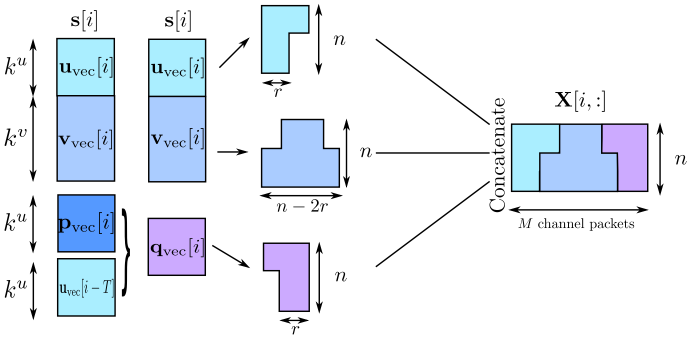
# Optimal Code Construction (cont.)

- Overall combined parity: $\mathbf{q}_{\text{vec}}[i] = \mathbf{p}_{\text{vec}}[i] + \mathbf{u}_{\text{vec}}[i - T]$



# Optimal Code Construction (cont.)

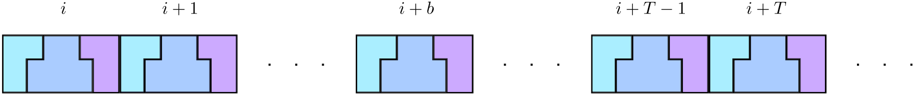
3. Mapping of macro-packet to individual channel-packets
   - Map the generated macro-packet into $M$ individual channel-packets




Rate of the code$=\frac{k_u+k_v}{2k_u+k_v}$

# Optimal Code Construction (cont.)

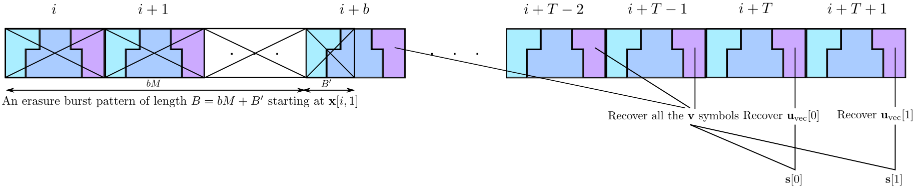
Overall macro-packet structure:




Overall macro-packet arrangment

# Decoding Analysis

Key Fact: The worst case burst pattern starts at the beginning of the macro-packet.




Let $B = bM + B'$. Two cases depending on whether $B' \lessgtr (1-R)M$:

Burst only erases symbols from $\mathbf{u}_{\text{vec}}[i+b]$

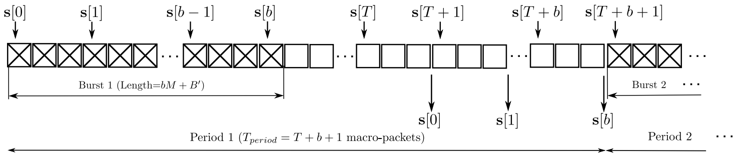
- Recovery of $\mathbf{v}$ symbols: $(k_u + k_v)b = k_u T$
- Optimal spitting ratio: $\frac{k_u}{k_v} = \frac{b}{T-b}$
- $R = \frac{T}{T+b}$

Burst erases symbols from $\mathbf{v}_{\text{vec}}[i+b]$

- Recovery of $\mathbf{v}$ symbols: $(k_u + k_v)b + (B'n - k_u) = k_u T$
- Optimal spitting ratio: $\frac{k_u}{k_v} = \frac{B}{M(T+b+1)-2B}$
- $R = \frac{M(T+b+1)-B}{M(T+b+1)}$

# Converse

- $T > b$: Technique of periodic erasure channel




  - Enough parities to recover from full burst $bM + B'$ $\implies$ $1 - R \geq \frac{bM+B'}{M(T+b+1)}$
  - Enough parities to recover from burst length $bM$ $\implies$ $1 - R \geq \frac{bM}{M(T+b)}$
- $T = b$: Periodic channel argument not tight, different argument

# Capacity Result

## Theorem

*For the streaming setup considered, with any $M$, $T$ and $B$, the streaming capacity $C$ is given by the following expression:*

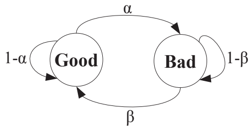
$$C = \begin{cases} \frac{T}{T+b}, & B' \le \frac{b}{T+b}M,\ \ T \ge b, \\ \frac{M(T+b+1)-B}{M(T+b+1)}, & B' > \frac{b}{T+b}M,\ \ T > b, \\ \frac{M-B'}{M}, & B' > \frac{M}{2},\ \ T = b, \\ 0, & T < b. \end{cases}$$

*where the constants $b$ and $B'$ are defined via*

$$B = bM + B', \quad B' \in \{0, 1, \ldots, M-1\}, \quad b \in \mathbb{N}^0.$$

□

# Simulation Result




## Gilbert Channel

- Bad state: $\Pr(\text{loss}) = 1$
- Good state: no losses

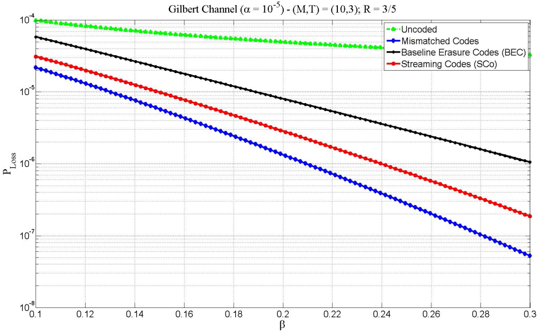


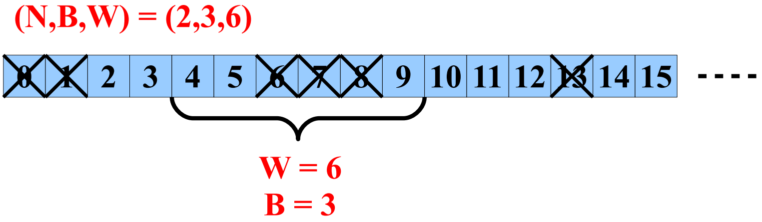

Figure: $(M, T, R) = (10, 3, 3/5)$

# Robust Extension

Extending the channel model to account for i.i.d. isolated erasures:

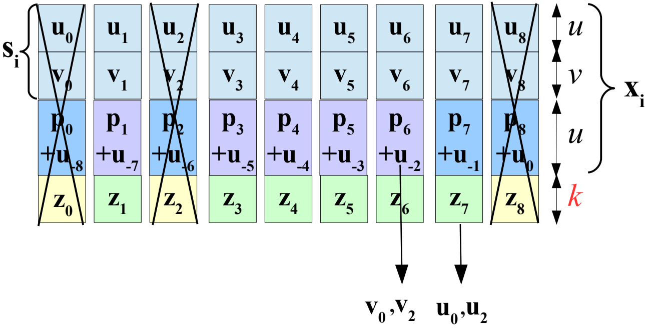
Any sliding window of length $W$ contains

- A **burst** of maximum length $B$, or,
- No more than $N$ erasures in **arbitrary** positions.



# Robust Extension for $M = 1$

- Append an additional layer parity checks containing Strongly-MDS code on **u**




- $R = \frac{u+v}{2u+v+k}$.
- Very close to being optimal for $k = \frac{N}{T-N+1}B$

# Robust Extension for any M

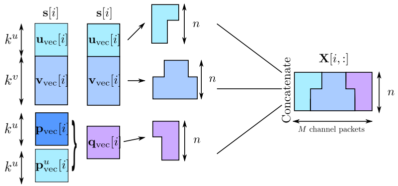
Many possibilities for the placement of Strongly-MDS parities for $u$!

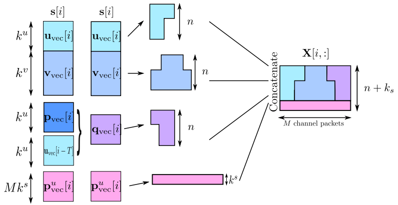
Approach I:

- Repetition code replaced by Strongly-MDS code for $\mathbf{u}$
- Rate of the code unchanged.
- $N = r$ is achievable.

Approach II

- Append additional layer of Strongly-MDS code for $\mathbf{u}$
- $R = \frac{k_u + k_v}{2k_u + k_v + k_s}$.
- $k_s$ chosen according to given $N$.



# Conclusions

1. Extension of the previously studied streaming setup ($M = 1$) to any general $M$ (source-channel rate mismatch).
2. Capacity characterization for a certain class of burst-erasure channels.
3. Layered code construction approach.
4. Robust code extension for protection against i.i.d. isolated erasures.

Thanks for listening!

Any questions/comments/thoughts?