

Confidence Intervals for 1:1 Matching Tasks

Riccardo Fogliato

arXiv: <https://arxiv.org/abs/2306.01198>

GitHub: <https://github.com/aws-labs/cis-matching-tasks>

Background

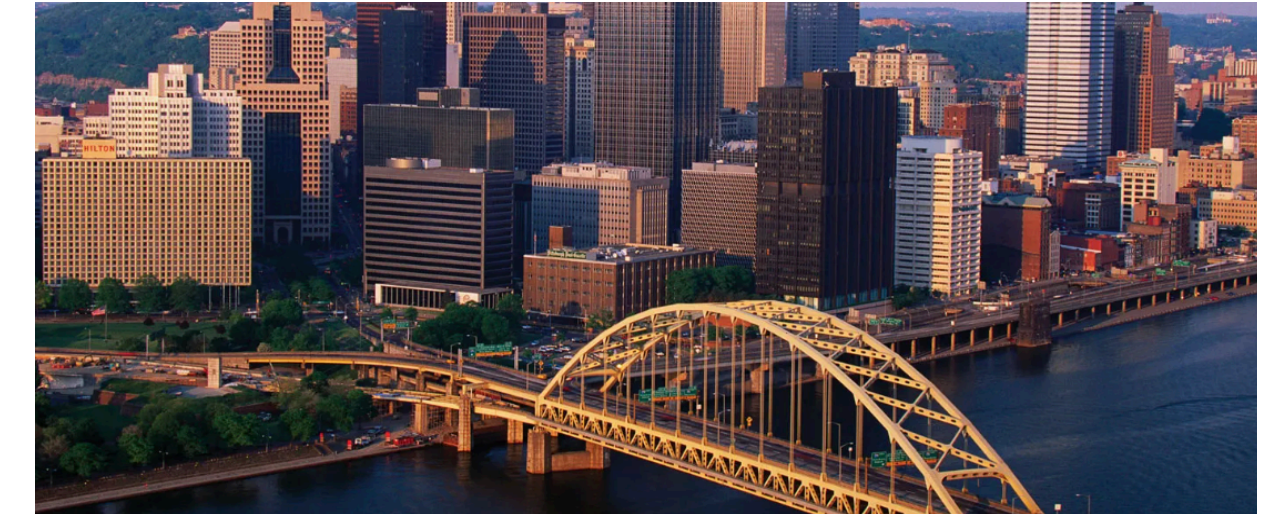
Undergrad @ Uni Padova



Master @ Uni Torino



PhD @ CMU



Applied Scientist @ AWS



Joint work with



Pratik Patil
UC Berkeley



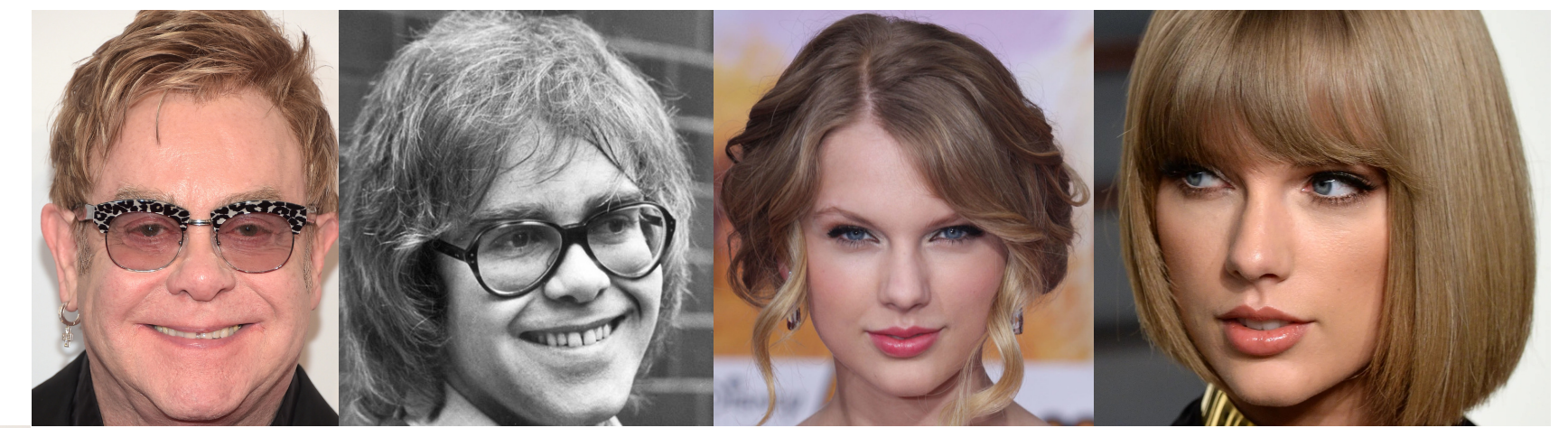
Pietro Perona
Caltech & AWS

1:1 matching tasks involve comparing two items to verify a match

Are the people in the two images
the same person or not?

Y: Same person

N: Different person



Y N N

N N

Y

Confidence Intervals for 1:1 Matching Tasks

Why do we need uncertainty?

TPM

Can you check the performance of our facial recognition service on this customer's data?

Scientist

Sure! Let me generate the predictions

Why do we need uncertainty?

TPM

Can you check the performance of our facial recognition service on this customer's data?

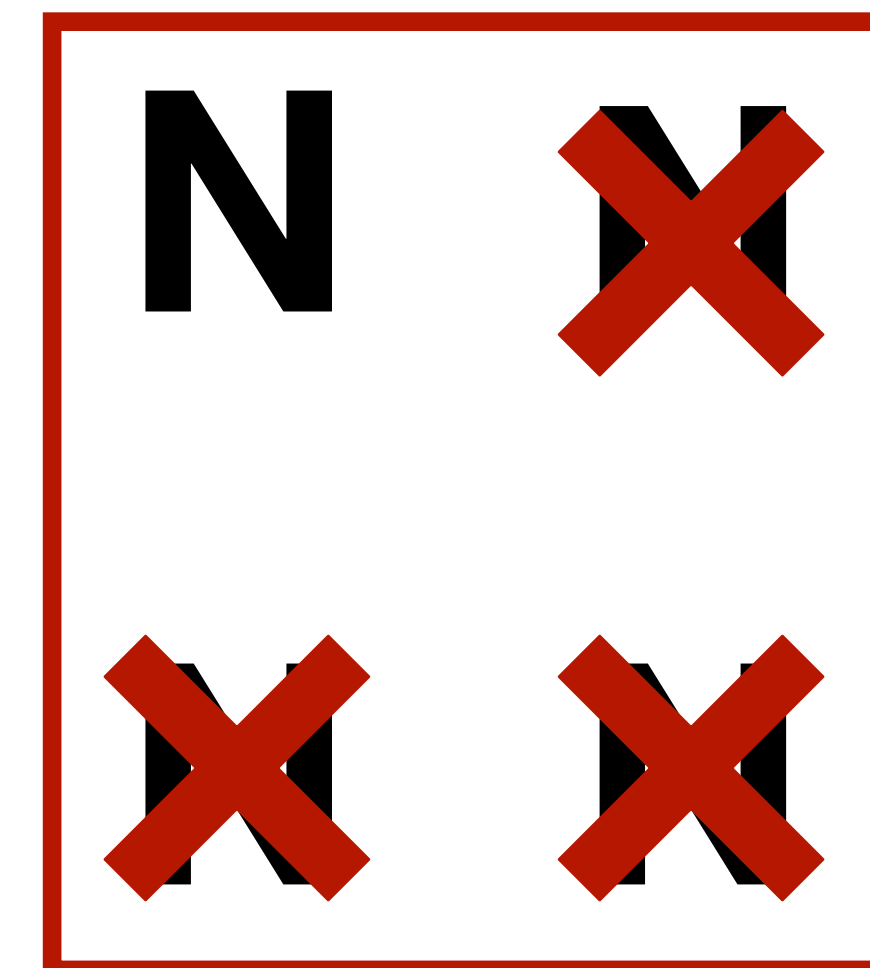
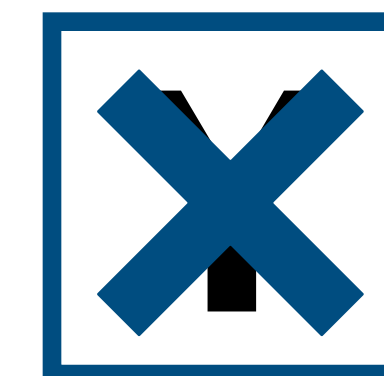
Scientist

Sure! Let me generate the predictions

Here are the results!
False Accept Rate (FAR) = 75%
False Reject Rate (FRR) = 50%

TPM

Oh no this is horrible!!!!



Why do we need uncertainty?

Scientist

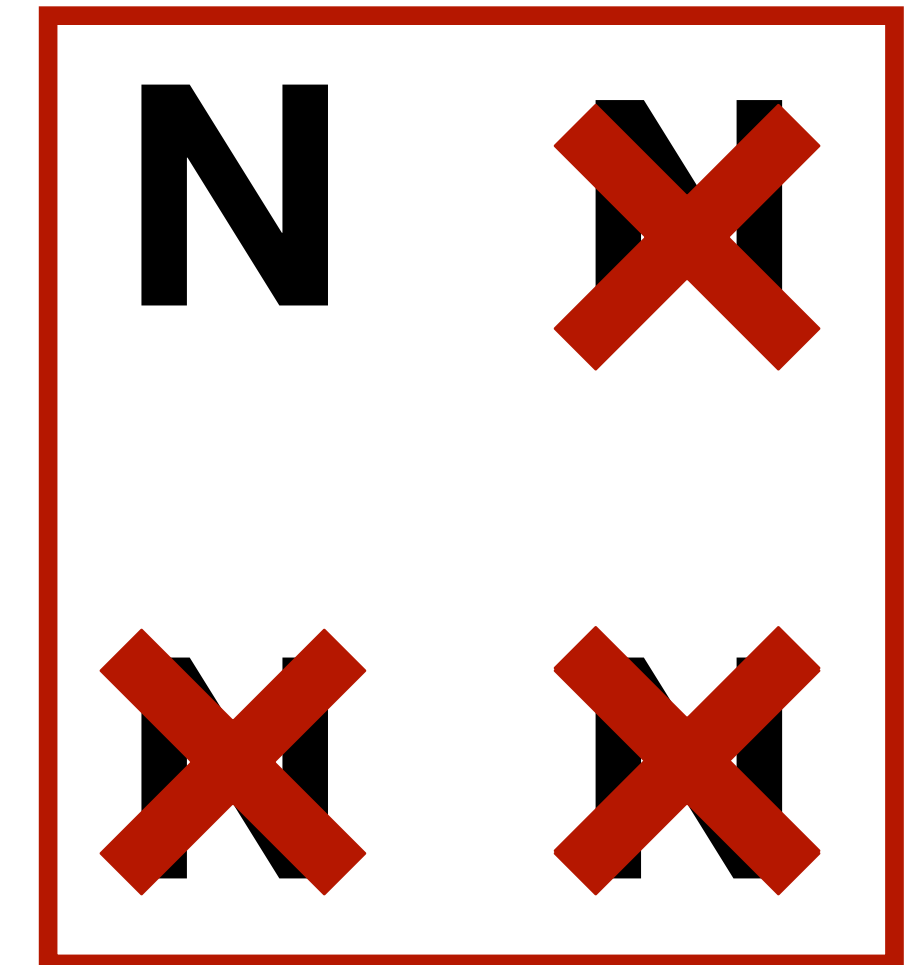
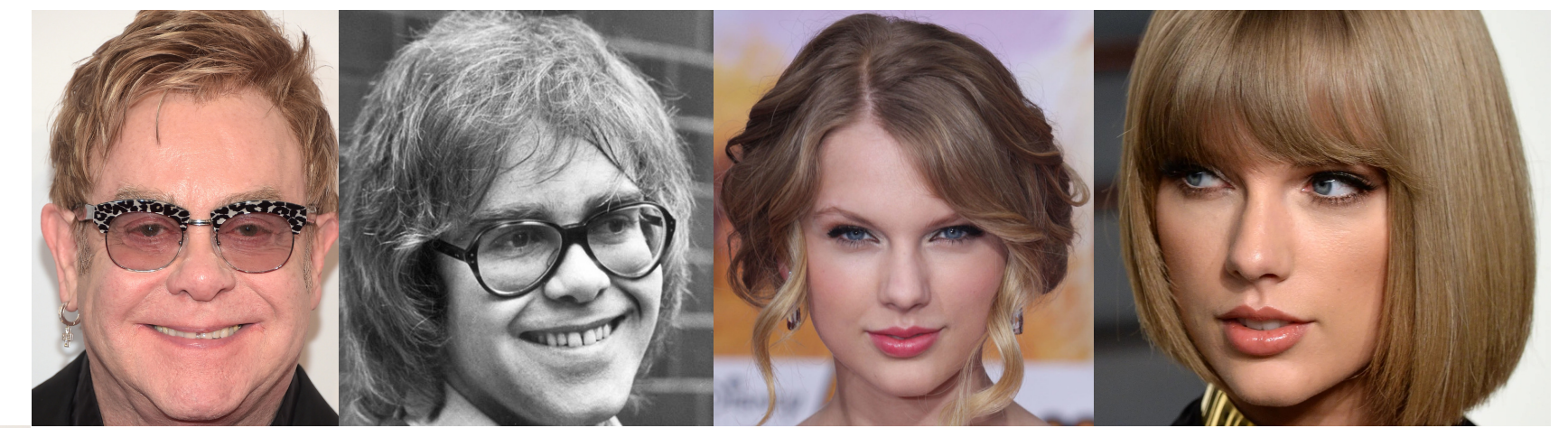
Oops I forgot the 95% confidence intervals!
False Accept Rate (FAR) = 75% (10%, 80%)
False Reject Rate (FRR) = 50% (10%, 60%)

So much uncertainty...

TPM

What a relief...!

We need higher standards! We'll make reporting uncertainty estimates mandatory from now on.



**How do we construct
confidence intervals
in 1:1 matching tasks?**

Commonly used approaches

Wald and (naive) Wilson intervals based on the Normal approximation of the maximum likelihood estimator

Assumptions:

- Independent data
- Identically distributed data
- Finite mean and variance
- Large sample size



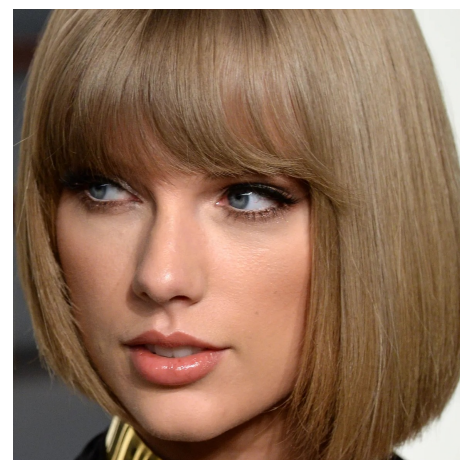
Standard central limit theorem assumptions do not hold in the context of 1:1 matching tasks

Assumptions:

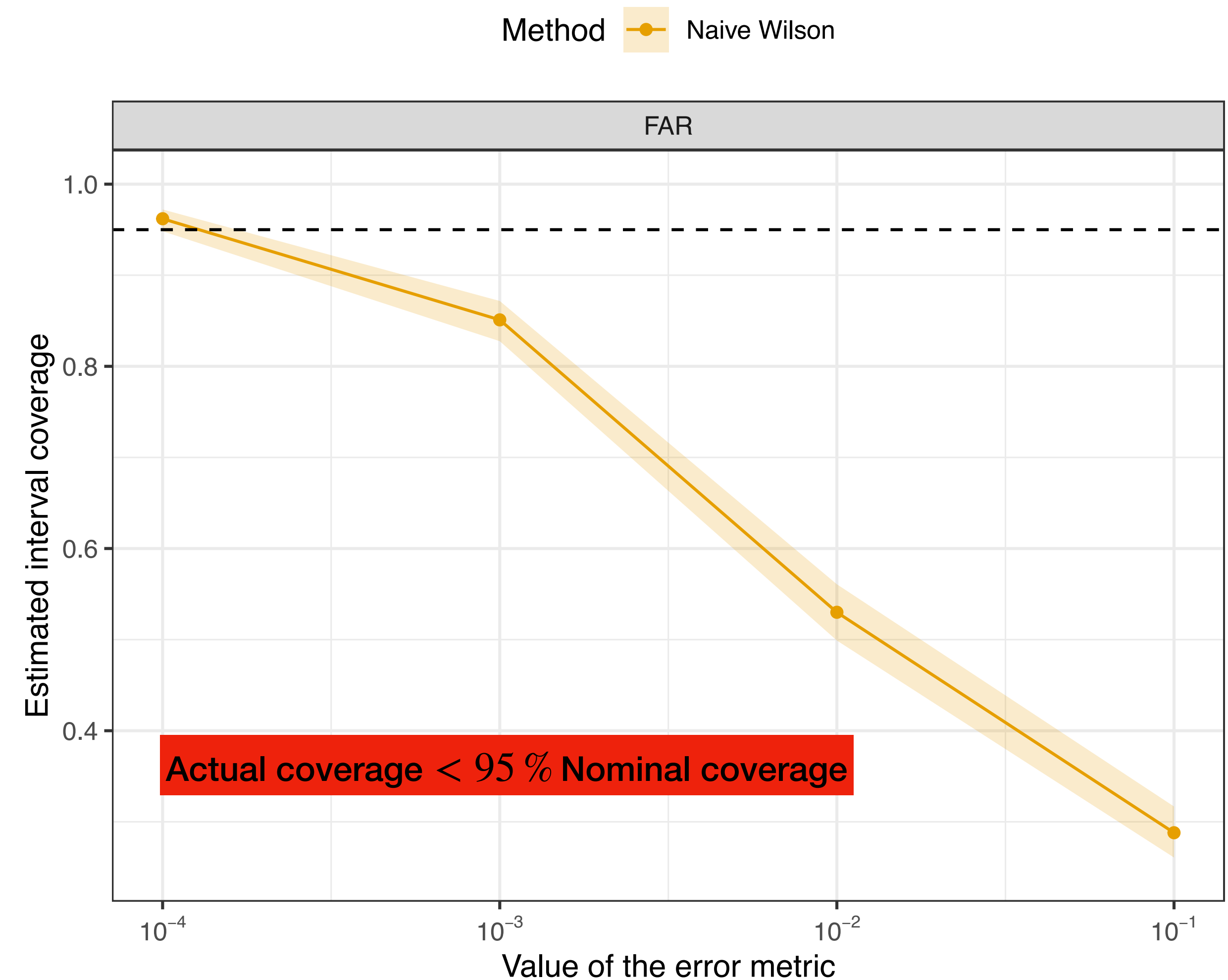
- ~~Independent data~~
- Identically distributed data
- Finite mean and variance
- Large sample size



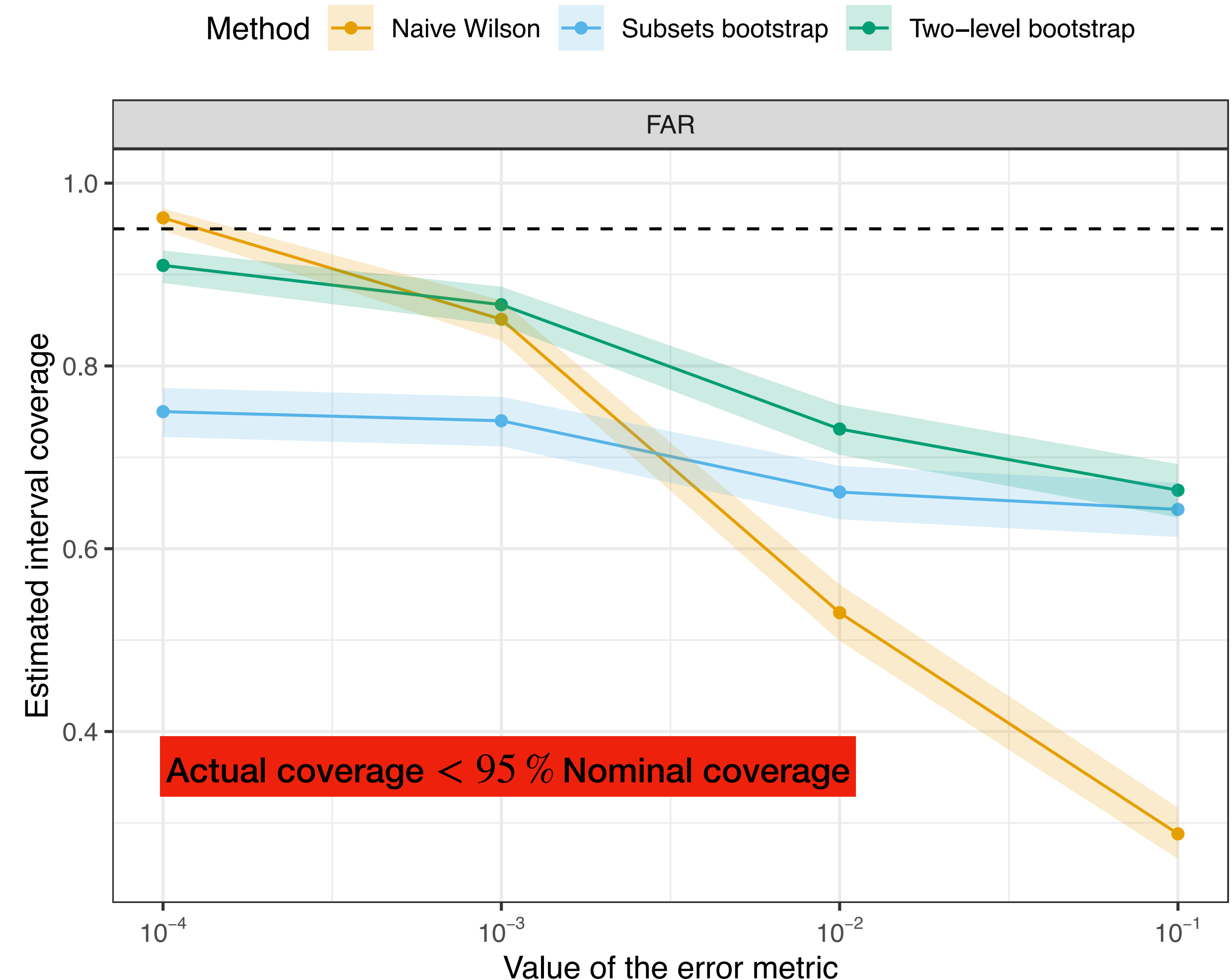
vs.



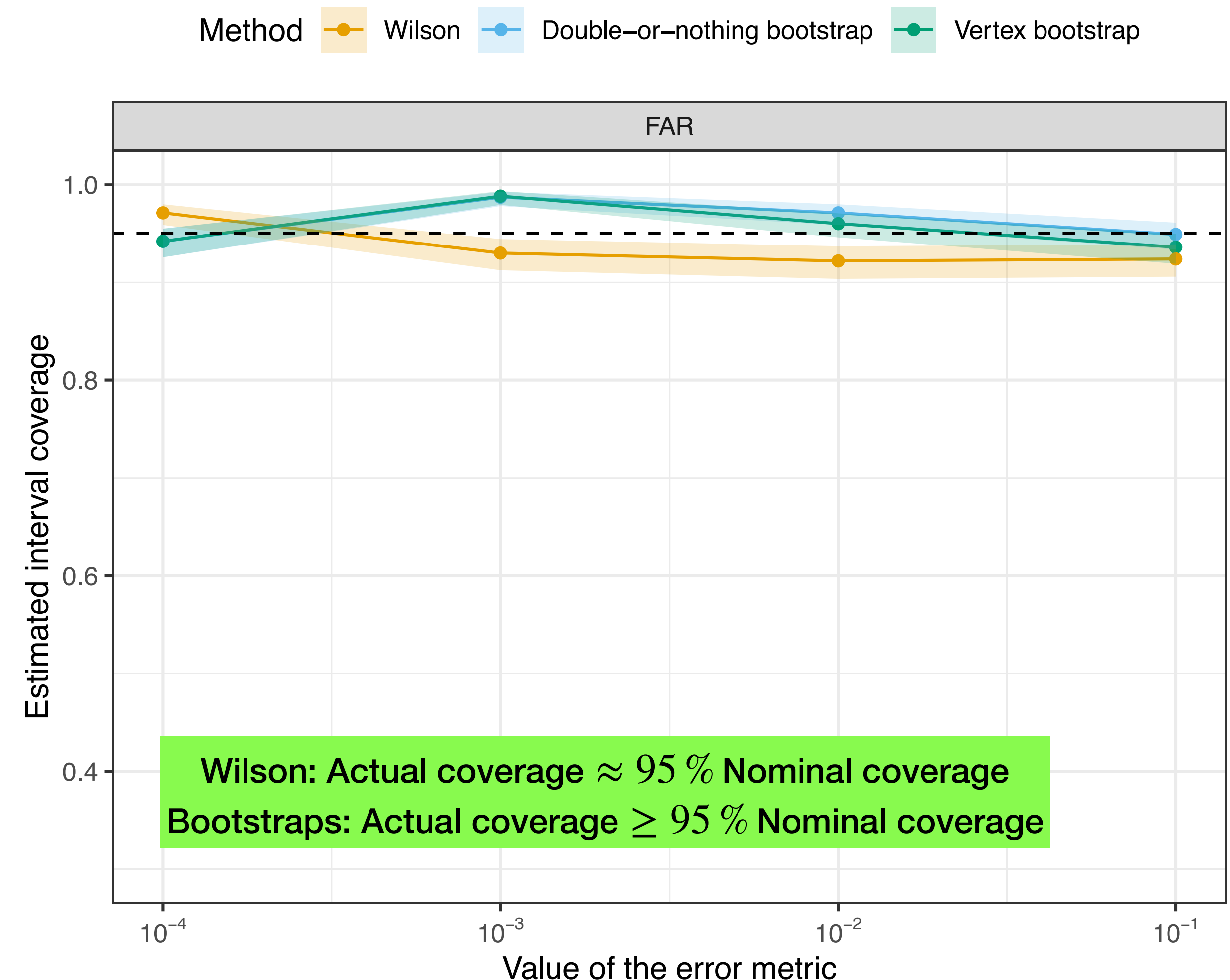
Naive Wilson intervals for the FAR are too narrow



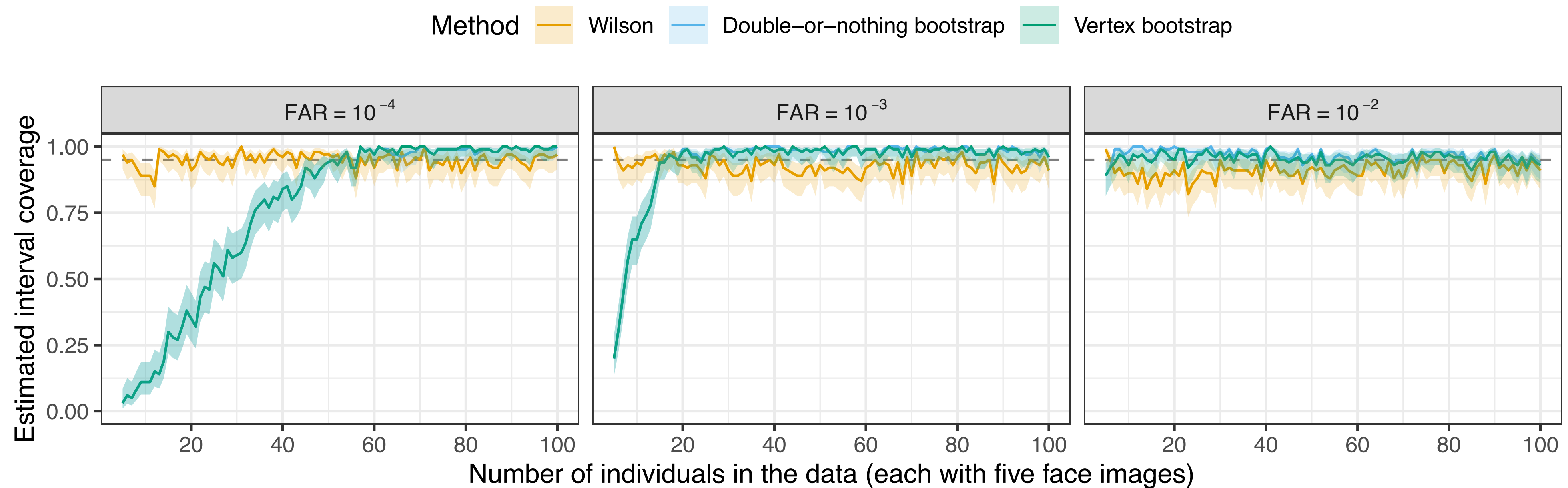
Bootstrap methods used in Facial Recognition produce FAR intervals that are too narrow



**(Improved) Wilson,
double-or-nothing
and vertex bootstrap
produce FAR intervals
that mostly achieve
nominal coverage**



Bootstrap intervals are inadequate when error rates are too small



Wilson: Actual coverage \approx 95 % Nominal coverage
Bootstraps: Actual coverage \geq 95 % Nominal coverage* when FAR \gg 0

How do we construct intervals that achieve nominal coverage for FAR in 1:1 matching tasks?

Constructing Wilson intervals

Naive and (correct) Wilson intervals for FAR are given by

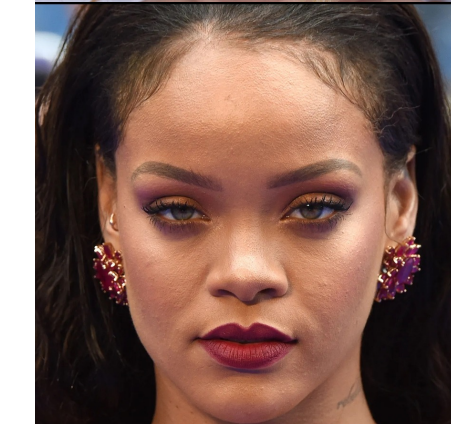
$$\frac{\widehat{FAR} \hat{N}_{FAR}^* + \frac{1}{2} z_{1-\alpha/2}^2}{\hat{N}_{FAR}^* + z_{1-\alpha/2}^2} \pm \frac{z_{1-\alpha/2} \sqrt{\hat{N}_{FAR}^*}}{\hat{N}_{FAR}^* + z_{1-\alpha/2}^2} \sqrt{\widehat{FAR} (1 - \widehat{FAR}) + z_{1-\alpha/2}^2 / (4 \hat{N}_{FAR}^*)}$$

where $\hat{N}_{FAR}^* = (\widehat{FAR} (1 - \widehat{FAR})) / \text{Var}(\widehat{FAR})$

(Correct) Wilson intervals

$$\text{Var}(\widehat{FAR}) = \frac{1}{3} \left[\text{Var}(\widehat{FAR}_{12}) + \text{Var}(\widehat{FAR}_{13}) + \text{Var}(\widehat{FAR}_{23}) \right] \text{ Naive Wilson}$$

$$+ \frac{2}{3} \left[\text{Cov}(\widehat{FAR}_{12}, \widehat{FAR}_{13}) + \text{Cov}(\widehat{FAR}_{12}, \widehat{FAR}_{23}) + \text{Cov}(\widehat{FAR}_{13}, \widehat{FAR}_{23}) \right]$$



\widehat{FAR}_{12}

\widehat{FAR}_{13}

\widehat{FAR}_{23}

Constructing double-or-nothing bootstrap intervals

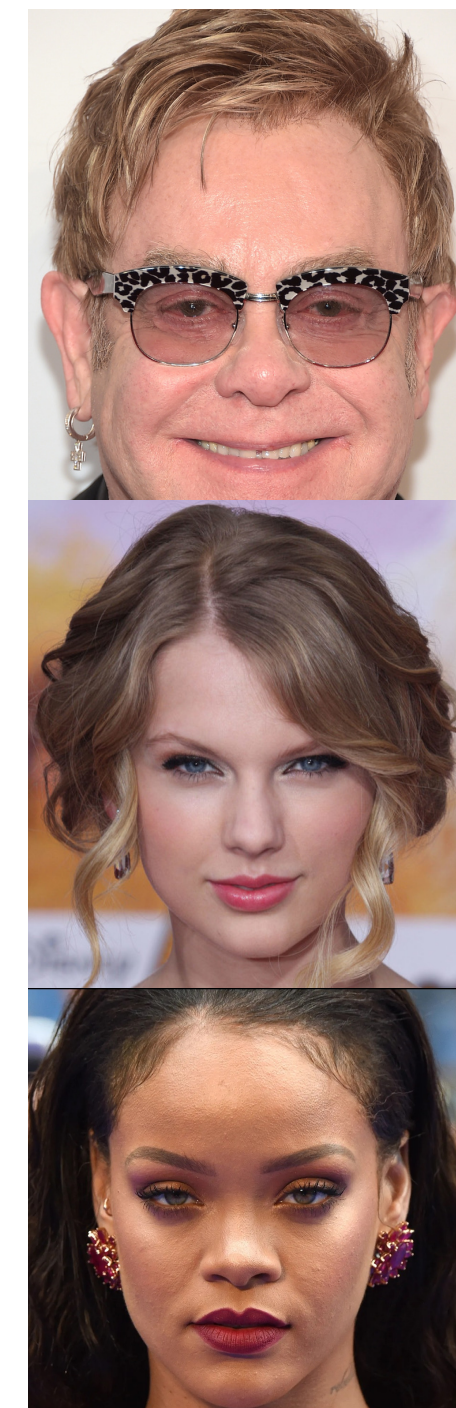
Percentile bootstrap recipe for $1 - \alpha$ FAR intervals

For repetition $b = 1, \dots, B$:

- sample $w_i \sim \text{Uniform}\{0,2\}$ for $i = 1, \dots, G$ and compute

$$\widehat{\text{FAR}}^b = \frac{\sum_{i \neq j} w_i w_j \widehat{\text{FAR}}_{ij}}{\sum_{i \neq j} w_i w_j}$$

Then take the $(\alpha/2, 1 - \alpha/2)$ quantiles of the $\widehat{\text{FAR}}_b$ estimates



b=1

b=2

b=B

$$w_1 = 2$$

$$w_1 = 0$$

$$w_1 = 2$$

$$w_2 = 2$$

$$w_2 = 2$$

$$w_2 = 2$$

$$w_3 = 0$$

$$w_3 = 2$$

$$w_3 = 2$$

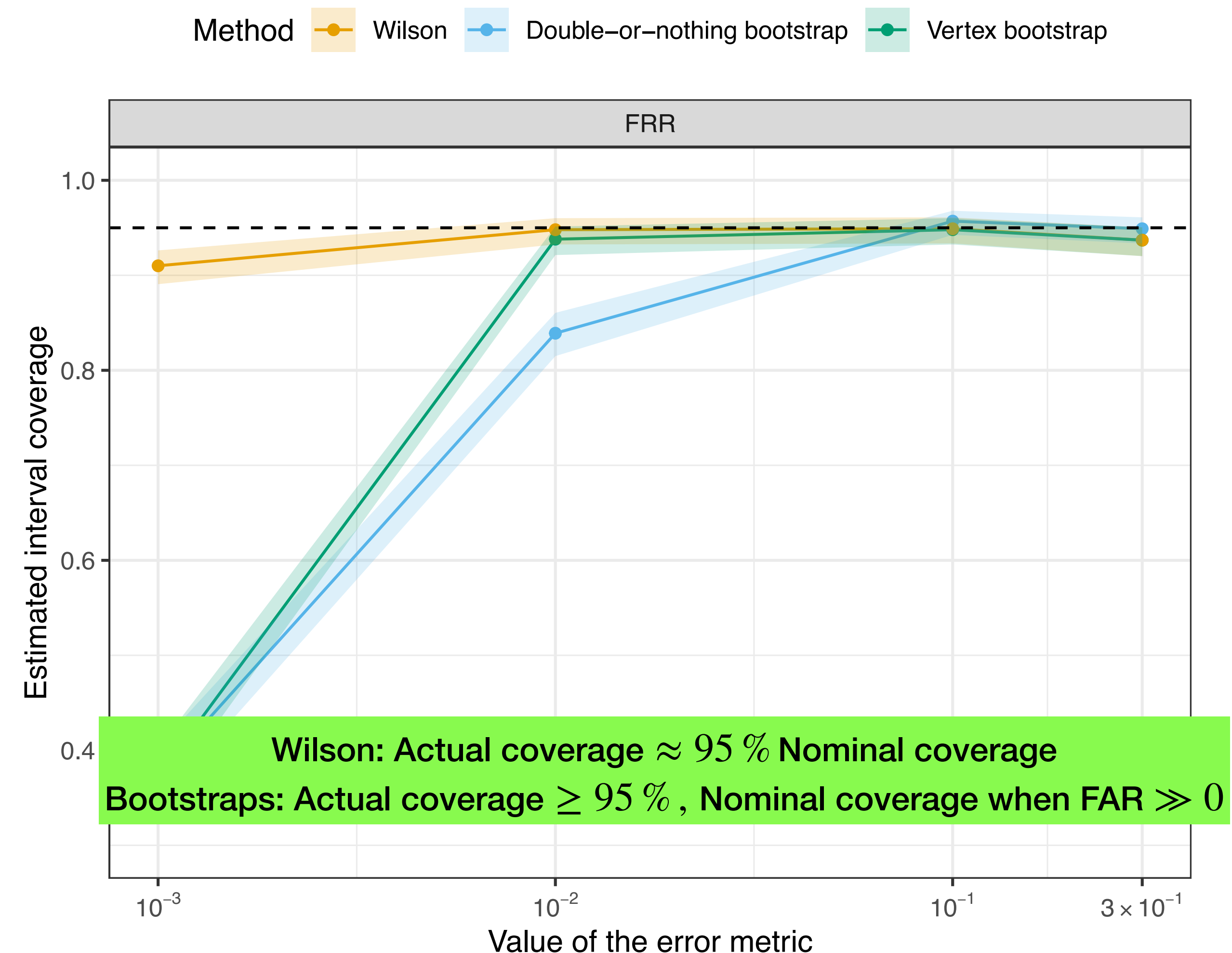
$$\widehat{\text{FAR}}^1 = \widehat{\text{FAR}}_{12}$$

$$\widehat{\text{FAR}}^2 = \widehat{\text{FAR}}_{23}$$

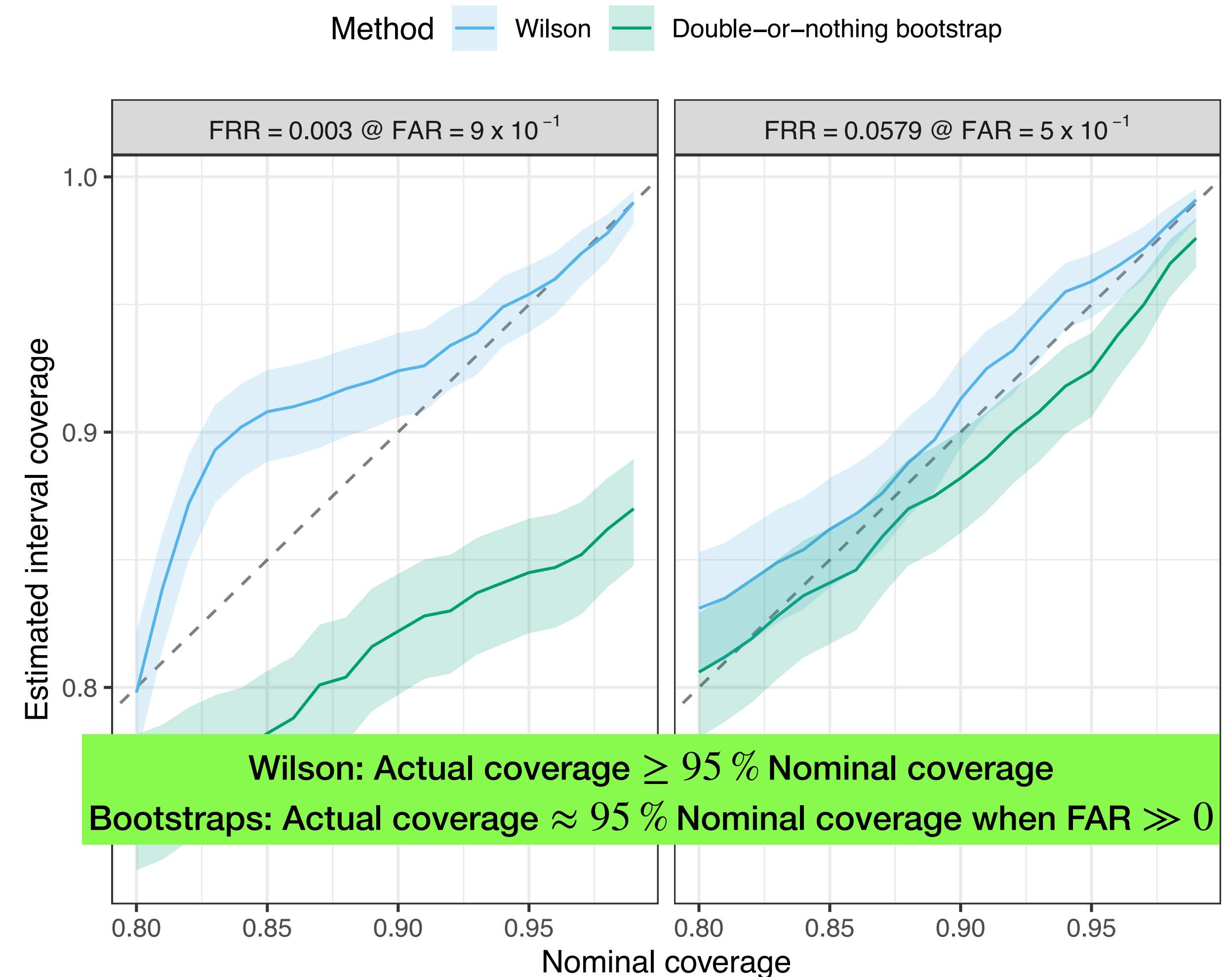
$$\widehat{\text{FAR}}^B = \widehat{\text{FAR}}$$

**How do we construct intervals
that achieve nominal coverage
for FRR and ROC
in 1:1 matching tasks?**

**Wilson,
double-or-nothing, and
vertex bootstrap
produce FRR intervals
that mostly achieve
nominal coverage**

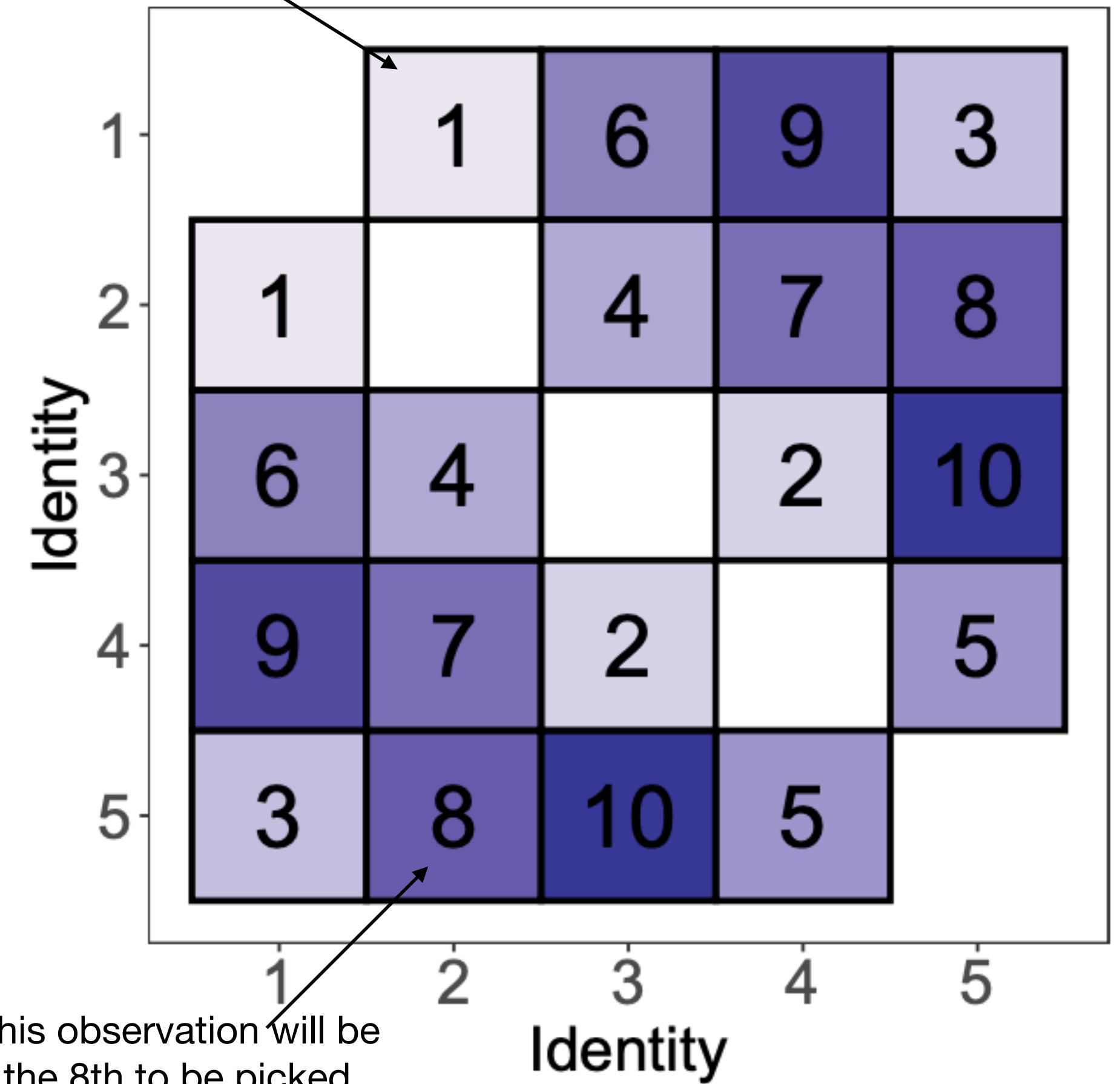


Wilson-based intervals for the ROC are conservative, while double-or-nothing bootstrap intervals achieve nominal coverage for larger error metrics



**Massive dataset and constrained resources?
To minimize the variance of FAR and FRR estimates, protocols should consider independent observations**

This observation will be the 1st to be picked in the protocol



This observation will be the 8th to be picked in the protocol

Takeaway for FAR/FRR intervals



Takeaway for ROC/AUC intervals

**Double or nothing
bootstrap and
vertex bootstrap**



Wilson intervals

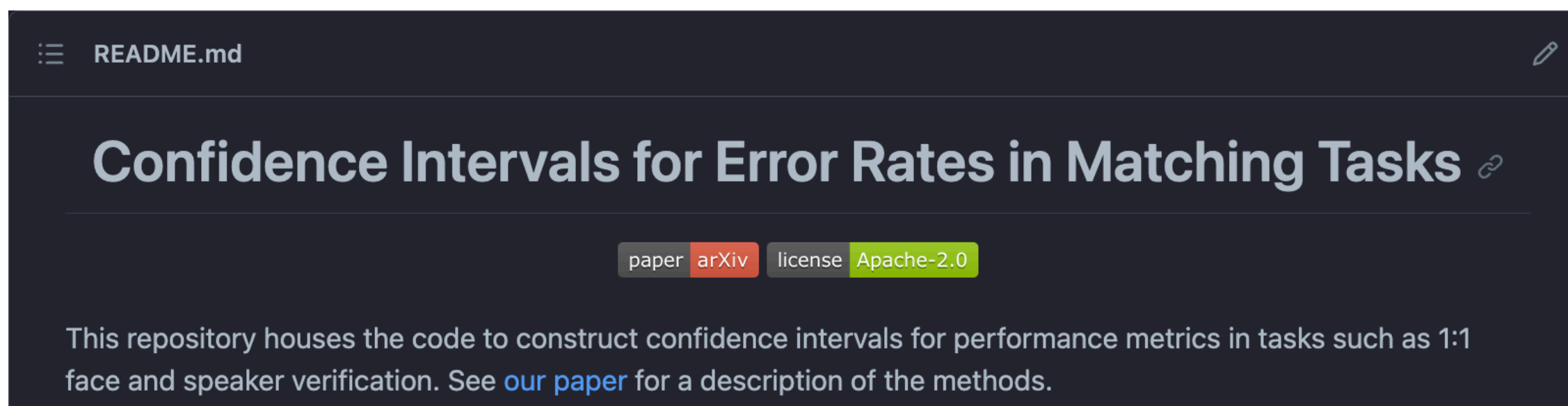


**Naive Wilson,
subsets, and two-
level bootstrap**



Code for reviewed methods: github.com/awslabs/cis-matching-tasks

General tutorials: github.com/awslabs/cis-matching-tasks

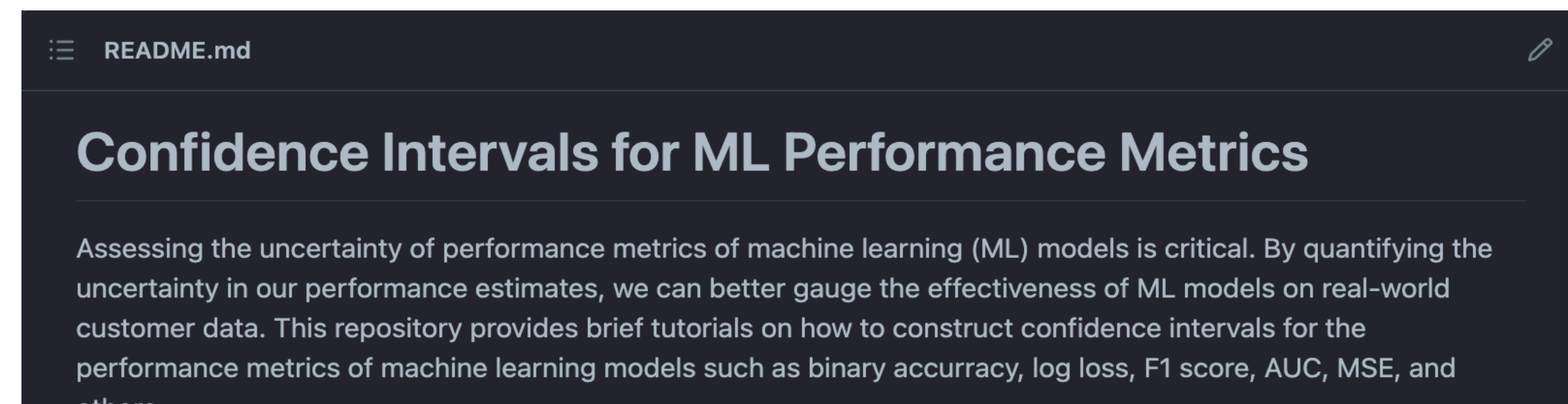


README.md

Confidence Intervals for Error Rates in Matching Tasks

paper arXiv license Apache-2.0

This repository houses the code to construct confidence intervals for performance metrics in tasks such as 1:1 face and speaker verification. See [our paper](#) for a description of the methods.



README.md

Confidence Intervals for ML Performance Metrics

Assessing the uncertainty of performance metrics of machine learning (ML) models is critical. By quantifying the uncertainty in our performance estimates, we can better gauge the effectiveness of ML models on real-world customer data. This repository provides brief tutorials on how to construct confidence intervals for the performance metrics of machine learning models such as binary accuracy, log loss, F1 score, AUC, MSE, and others.

Tutorial on MORPH

In this tutorial, we will assess the performance of a facial recognition system in a 1:1 face verification task on the [MORPH dataset](#). We have obtained the embeddings generated by the system for the images in the data and stored them in a dictionary `df[identity name][image name] = embedding`. Below we load the dictionary.

```
import json
from utils import *

df_main = json.load(open('../data/morph/embeddings.json', 'r'))
len(df_main) # number of identities in the data
```

63548

We analyze the system performance in two settings:

- small datasets:** We assess the system performance on all pairwise comparisons between the images in the data.
- large datasets:** We compute the system performance on a subset of all pairwise comparisons between the images in the data.

Classification Tasks

This notebook covers the construction of confidence intervals and hypothesis testing for metrics typically employed to evaluate the performance of ML models in binary classification tasks. These methods are model agnostic, in that they apply to any model that outputs a confidence score for each prediction.

Problem Setup

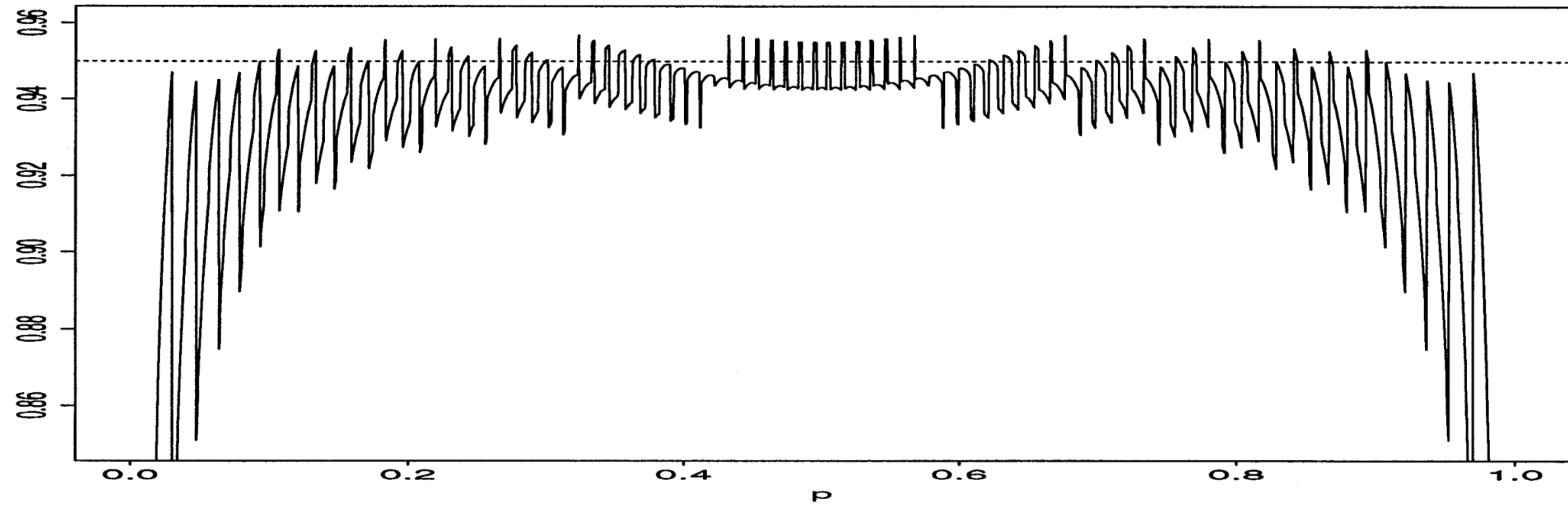
Overview

We have a dataset with n observations (X_i, Y_i) , where each pair is independently and identically distributed (IID) from a probability distribution P . Here, X_i is a vector of features, and Y_i is a binary outcome. The outcome Y is defined as:

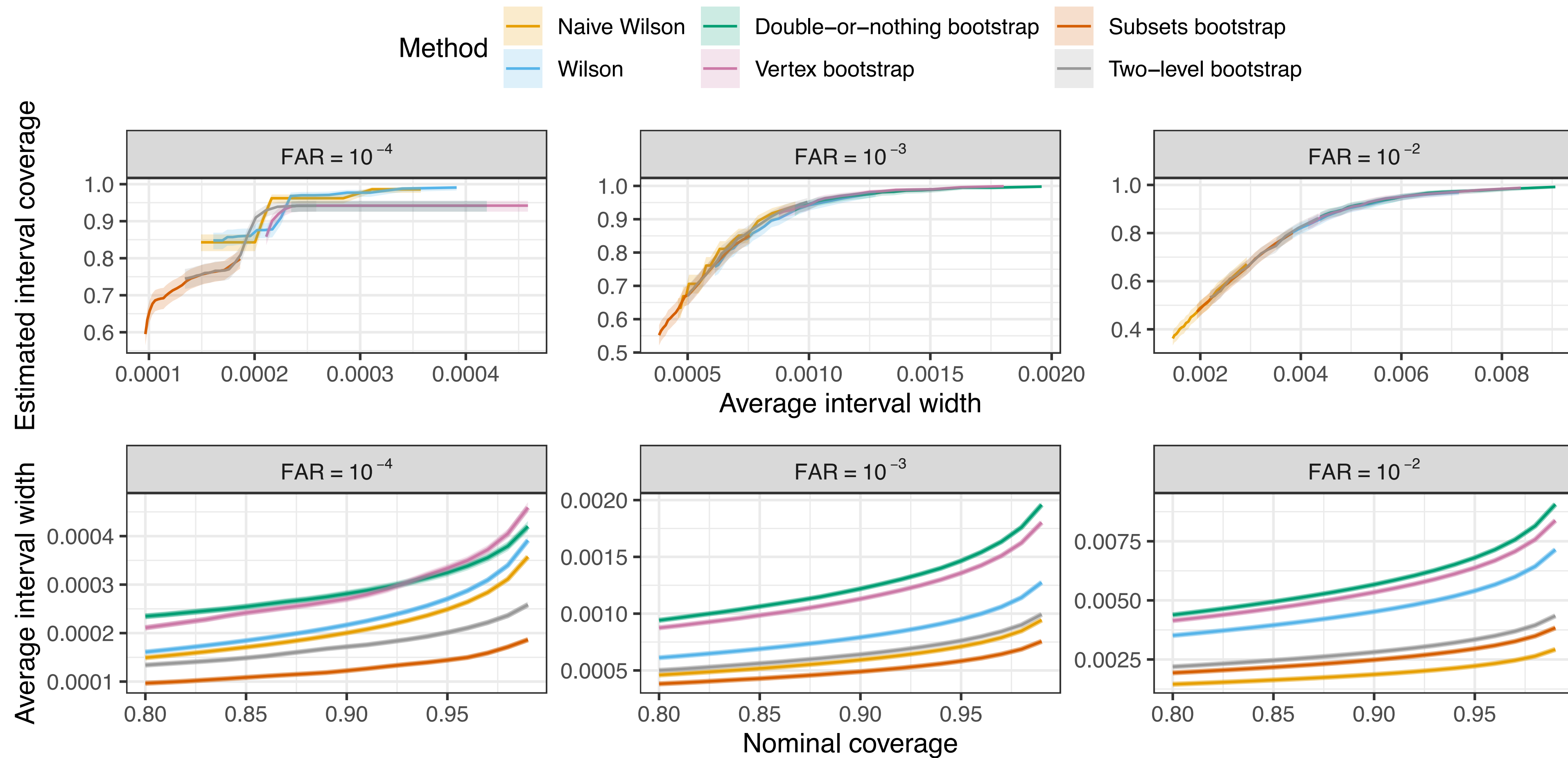
$$Y = \begin{cases} 1 & \text{with probability } \mathbb{E}_P[Y|X], \\ 0 & \text{with probability } 1 - \mathbb{E}_P[Y|X] \end{cases}$$

Thank you!

Wald intervals fail in 1:1 matching tasks when error rates are low



Width vs. coverage



Width of various algorithms

