Cloud Radio Access Networks: Capacity and Duality

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Future Cellular Wireless: Dense, Massive, and Cooperative

- Bottlenecks for cellular networks:
  - Path-loss, fading, and interference
- Emerging useful ideas:
  - Dense
    - Heterogeneous network
  - Massive
    - Large-scale MIMO in each base station (BS)
  - Cooperative
    - Signal processing for interference cancellation
- This talk: Cooperative communication

Cloud Radio Access Network (C-RAN)

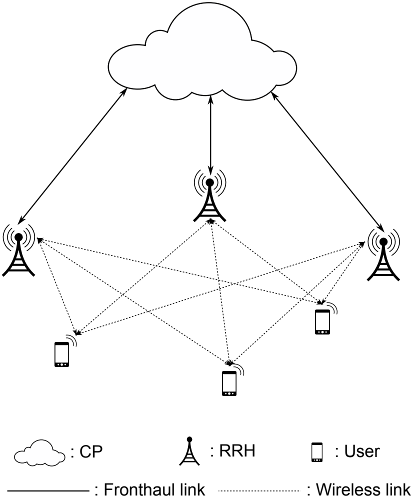


Figure: Illustration of the CRAN downlink

C-RAN Architecture

- C-RAN
  - BSs are connected to a centralized, cloud-computing based processor.
  - Backhaul links have high (but not infinite) capacities.
- Motivation
  - Centralized service provisioning, easy BS upgrade, etc.
  - Enable joint multi-cell processing interference management.
- Uplink
  - Joint decoding in the cloud.
  - Virtual multiple-access channel with BSs as relays.
- Downlink
  - Joint encoding in the cloud.
  - Virtual broadcast channel with BSs as relays.

Uplink C-RAN

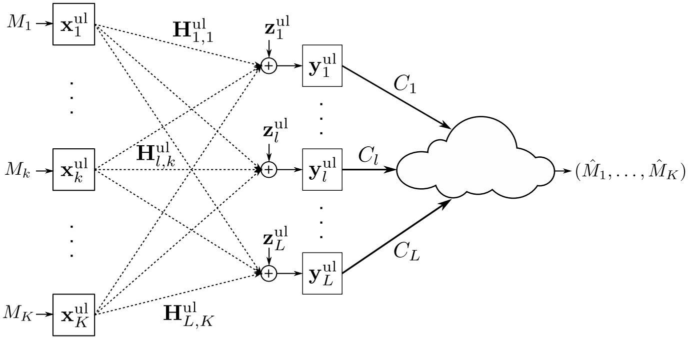


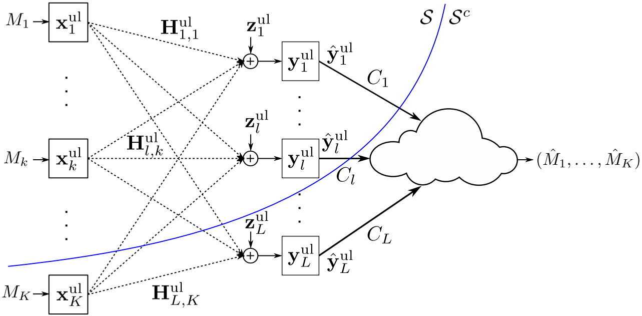
Figure: Information Theoretic Model

- Infinite backhaul case: Uplink C-RAN is just a MIMO multiple-access channel.
- This talk: Practical and more challenging case of finite backhaul.

Noisy Network Coding (NNC)

- A general coding scheme for multiple-access relay channels [Lim et al., 2011].
- Combines and generalizes network coding over noiseless networks and compress-forward for the 3-node relay channel.
- Key idea is to let the decoder perform *simultaneous* decoding of the received compressed signals *without* uniquely decoding the compression indices.
- For N-node Gaussian multiple-access-relay network, NNC achieves to within $0.63N$ bits from the cutset bound.
- Generalization of previous work on coding for relay network and deterministic networks [Avestimehr et al., 2011].

NNC applied to uplink C-RAN



- Cutset outer bound:
  $R(S) := \sum_{k \quad S} R_k \leq I(\mathbf{x}^{\mathrm{ul}}(S); \mathbf{y}^{\mathrm{ul}}(S^c) | \mathbf{x}^{\mathrm{ul}}(S^c)), \quad S$ for some $p(\mathbf{x}_{\mathrm{ul}}^N)$
- NNC inner bound:
  $R(S) \leq$
  $I(\mathbf{x}^{\mathrm{ul}}(S); \hat{\mathbf{y}}^{\mathrm{ul}}(S^c), \mathbf{y}_d^{\mathrm{ul}} | \mathbf{x}^{\mathrm{ul}}(S^c)) - I(\mathbf{y}^{\mathrm{ul}}(S); \hat{\mathbf{y}}^{\mathrm{ul}}(S) | \mathbf{x}_{\mathrm{ul}}^N, \hat{\mathbf{y}}^{\mathrm{ul}}(S^c), \mathbf{y}_d^{\mathrm{ul}}), \quad S$
  for some $\prod p(\mathbf{x}_k^{\mathrm{ul}}) p(\hat{\mathbf{y}}_k^{\mathrm{ul}} | \mathbf{y}_k^{\mathrm{ul}}, \mathbf{x}_k^{\mathrm{ul}})$
- Key idea: Choose $\hat{\mathbf{y}}_k^{\mathrm{ul}}$ close to $\mathbf{y}_k^{\mathrm{ul}}$ while controlling the penalty.

Downlink C-RAN

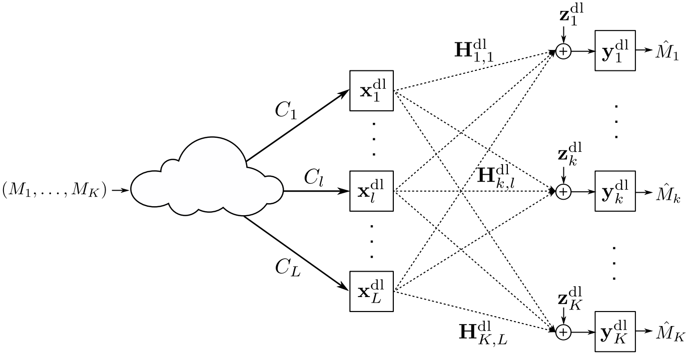


Figure: Information Theoretic Model

- Infinite backhaul case: Uplink C-RAN is just a MIMO broadcast channel.
- This talk: Practical and more challenging case of finite backhaul.

Existing Transmission Strategies for Downlink C-RAN

BSs need to broadcast: Beamforming + dirty paper coding
BSs also act as relays:

- Decode-and-forward relaying strategy (Data-sharing strategy):
  - User messages are shared with BSs for joint beamforming, e.g., [Marsch and Fettweis, 2009].
  - To limit backhaul, we need to form clusters [Ng et al., 2008], [Zakhour and Gesbert, 2011], [Zhao et al., 2013].
- Compression-and-forward relaying strategy (Compression-based strategy):
  - Precode at the cloud, compress the signals and send compressed versions to BSs. [Simeone et al., 2009], [Marsch and Fettweis, 2008].
  - Benefits of multivariate compression studied in [Park et al., 2013].
- Compute-and-forward relaying strategy [Nazer et al., 2009]:
  - Reverse-CoF and integer-forcing ideas studied [Hong and Caire, 2013].

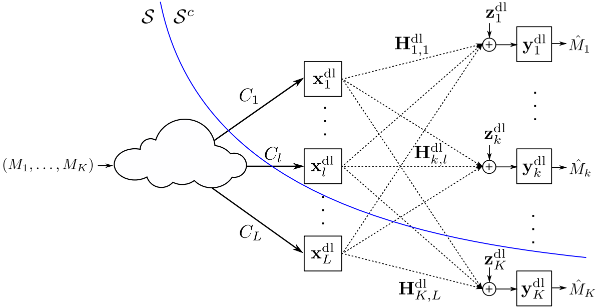
Distributed decode-forward (DDF)

- A general coding scheme for broadcast-relay channels [Lim et al., 2015].
- Combines and generalizes Marton coding for broadcast channel and partial decode-forward for 3-node relay channel.
- Key idea is to *apriori* precode all the codewords of the entire network at the source using multicoding. The codewords carry partial information about the destination messages *implicitly*.
- For N-node Gaussian broadcast-relay network, DDF achieves to within $0.5N$ bits from the cutset bound.
- Extension of previous work on approximately optimal broadcasting [Kannan et al., 2012].

DDF applied to downlink C-RAN

- DDF uses auxiliary codewords and indices, rather than explicit message splittings, for relays to decode and forward. The auxiliary codewords and the actual transmitted codewords are selected at the source to be entangled according to a specified joint distribution using multicoding.
- DDF applies to the downlink C-RAN model directly.
- A specific choice of the joint distribution of auxiliary and actual codewords can be shown to be within a constant gap from the cutset bound for downlink C-RAN.
    - For the relays at the output of the digital hop, the auxiliary codewords are chosen to be exactly same as the digital outputs (since these are noiseless links).
    - For the destinations at the output of the analog hop, the auxiliary codewords are chosen to be close the noisy outputs of the channel with independent noises.

DDF applied to downlink C-RAN

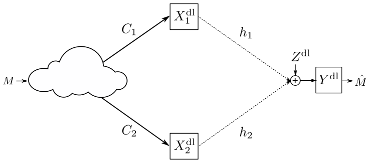


- Cutset outer bound:
  $R(S) \leq I(\mathbf{x}^{\mathrm{dl}}(S); \mathbf{y}^{\mathrm{dl}}(S^c) | \mathbf{x}^{\mathrm{dl}}(S^c)), \quad S$ for some $p(\mathbf{x}_{\mathrm{dl}}^N)$
- DDF inner bound:
  $R(S) \leq I(\mathbf{x}^{\mathrm{dl}}(S); \mathbf{u}(S^c) | \mathbf{x}^{\mathrm{ul}}(S^c)) -$
  $\sum_{k \ S^c} [I(\mathbf{u}_k^{\mathrm{dl}}; \mathbf{u}(S_k^c), \mathbf{x}_{\mathrm{dl}}^N | \mathbf{x}_k^{\mathrm{dl}}, \mathbf{y}_k^{\mathrm{dl}}) + I(\mathbf{x}_k^{\mathrm{dl}}; \mathbf{x}^{\mathrm{dl}}(S_k^c))], \quad S$ for some $p(\mathbf{x}_{\mathrm{dl}}^N, \mathbf{u}^N)$
- Key idea: Choose $\mathbf{u}_k$ close to $\mathbf{y}_k^{\mathrm{dl}}$ while controlling the penalty.

Relationship of DDF and compression-like strategy

- Compression-like strategy in downlink C-RAN can be obtained as a special case of DDF for a particular choice joint distribution of auxiliary and actual transmitted codewords.
- Is compression-like strategy universally within constant gap from the capacity for downlink C-RAN?
  - For some special cases, the answer is yes!
  - For the general downlink C-RAN model, the work is ongoing.

Relationship of DDF and compression-like strategy



Compression-like scheme

$$\begin{aligned} R &< I(U; Y^{\mathrm{dl}}) \\ C_1 &> I(U; X_1^{\mathrm{dl}}) \\ C_2 &> I(U; X_2^{\mathrm{dl}}) \\ C_1 + C_2 &> I(U; X_1^{\mathrm{dl}}) + I(U; X_2^{\mathrm{dl}}) \\ &\quad + I(X_1^{\mathrm{dl}}; X_2^{\mathrm{dl}}) \end{aligned}$$

evaluated over $p(u, x_1^{\mathrm{dl}}, x_2^{\mathrm{dl}})$

DDF

$$\begin{aligned} R < I(U; Y^{\mathrm{dl}}) + \min\{0, \\ C_1 - I(U; X_1^{\mathrm{dl}}), \\ C_2 - I(U; X_2^{\mathrm{dl}}), \\ C_1 + C_2 - I(U; X_1^{\mathrm{dl}}) - I(U; X_2^{\mathrm{dl}}) \\ - I(X_1^{\mathrm{dl}}; X_2^{\mathrm{dl}})\} \end{aligned}$$

evaluated over $p(u, x_1^{\mathrm{dl}}, x_2^{\mathrm{dl}})$

Uplink versus Downlink C-RAN

Uplink

- Multiple-access-relay channel
- Simple encoders, complex cloud decoder
- Compress-forward with independent or Wyner-Ziv compression
- Noisy network coding within constant gap

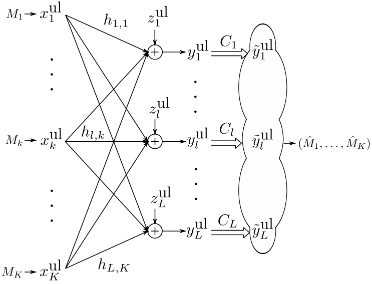
Downlink

- Broadcast-relay channel
- Simple decoders, complex cloud encoder
- Compression-like partial decode-forward with independent or multivariate compression covering
- Distributed decode-forward within constant gap

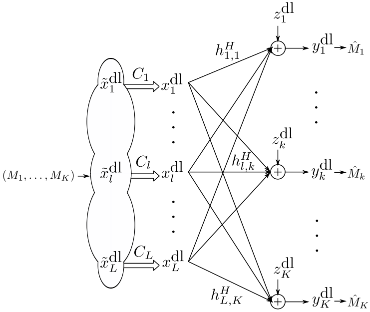
Dual relationship between uplink and downlink C-RAN

- If the fronthaul capacity is *infinite*:
  - uplink C-RAN becomes a MIMO multiple-access channel (MAC)
  - downlink C-RAN becomes a MIMO broadcast channel (BC)
- Gaussian MIMO MAC and BC channels have a dual relationship
  - Under linear beamforming and the same sum-power constraint, the achievable rate-regions are the same [Rashid-Farrokhi et al., 1998]
  - Under successive decoding (uplink) and dirty paper coding (downlink) and the same sum-power constraint, the capacity regions are the same [Viswanath and Tse, 2003], [Jindal et al., 2004]
  - Above results can be extended for per-terminal power constraints [Yu and Lan, 2007]
- Does similar duality relationship extend for uplink and downlink C-RAN with *finite* fronthaul capacities? Under what conditions?
  - Yes! For compress-forward in the uplink and compression-like scheme in the downlink with independent compression in both cases.

Uplink-Downlink Duality in C-RAN



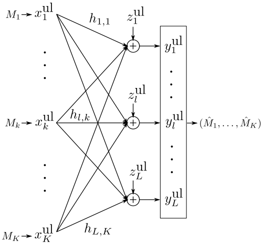
(a) Uplink



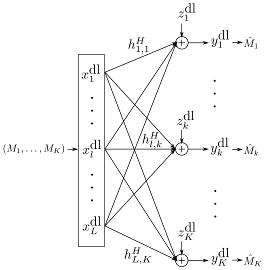
(b) Downlink

- With finite fronthaul capacities:
  - Uplink C-RAN: multiple-access relay channel
  - Downlink C-RAN: broadcast relay channel
- With infinite fronthaul capacities:
  - Uplink C-RAN   multiple-access channel (MAC)
  - Downlink C-RAN   broadcast channel (BC)

Uplink-Downlink Duality in C-RAN



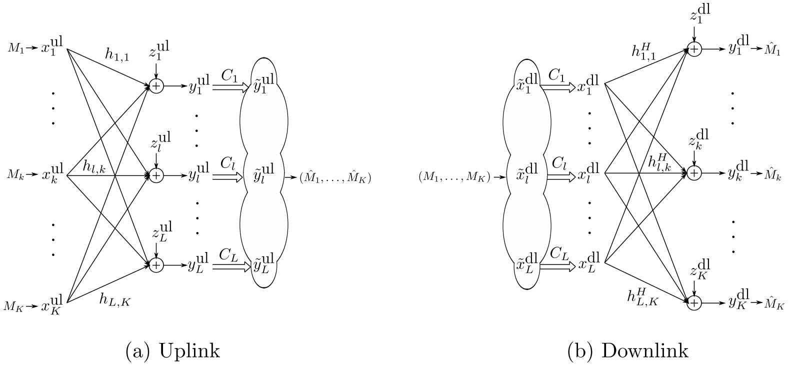
(a) Multiple Access Channel



(b) Broadcast Channel

- An uplink-downlink duality holds between MAC and BC
- With conjugate transpose channels and identical sum-power constraints:
  - Linear encoding and decoding: identical achievable rate regions [Rashid-Farrokhi et al., 1998]
  - Dirty paper coding and successive interference cancellation: identical capacity regions [Viswanath and Tse, 2003], [Jindal et al., 2004]

Uplink-Downlink Duality in C-RAN



- Is the duality true with finite fronthaul capacities?
- It is true if
  - Compression strategy is used in both the uplink and downlink;
  - All the RRHs and users are equipped with one antenna;
  - Independent compression is performed across RRHs.

Uplink-Downlink Duality in C-RAN

- Uplink
  - Transmit signal $x_k^{\mathrm{ul}} = \sqrt{p_k^{\mathrm{ul}}} s_k^{\mathrm{ul}}$
  - Receive beamforming $\tilde{s}_k^{\mathrm{ul}} = \mathbf{w}_k^H \tilde{\mathbf{y}}^{\mathrm{ul}}$
  - Total transmit power

$$P^{\mathrm{ul}}(\{p_i^{\mathrm{ul}}\}) = \sum_{i=1}^{K} p_i^{\mathrm{ul}}$$

  - Fronthaul link rate

$$C_l^{\mathrm{ul}}(\{p_i^{\mathrm{ul}}\}, q_l^{\mathrm{ul}}) = \log_2 \frac{\sum_{i=1}^{K} p_i^{\mathrm{ul}} |h_{l,i}|^2 + q_l^{\mathrm{ul}} + \sigma^2}{q_l^{\mathrm{ul}}}$$

  - End-to-end rate

$$R_k^{\mathrm{ul}}(\{p_i^{\mathrm{ul}}, \mathbf{w}_i\}, \{q_l^{\mathrm{ul}}\}) = \log_2 \frac{\sum_{i=1}^{K} p_i^{\mathrm{ul}} |\mathbf{w}_k^H \mathbf{h}_i|^2 + \sum_{l=1}^{L} q_l^{\mathrm{ul}} |w_{k,l}|^2 + \sigma^2}{\sum_{j=k} p_j^{\mathrm{ul}} |\mathbf{w}_k^H \mathbf{h}_j|^2 + \sum_{l=1}^{L} q_l^{\mathrm{ul}} |w_{k,l}|^2 + \sigma^2}$$

Uplink-Downlink Duality in C-RAN

- Downlink
  - Transmit signal

$$\begin{bmatrix} x_1^{\mathrm{dl}} \\ \vdots \\ x_L^{\mathrm{dl}} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{K} v_{i,1} \sqrt{p_i^{\mathrm{dl}}} s_i^{\mathrm{dl}} \\ \vdots \\ \sum_{i=1}^{K} v_{i,L} \sqrt{p_i^{\mathrm{dl}}} s_i^{\mathrm{dl}} \end{bmatrix} + \begin{bmatrix} e_1^{\mathrm{dl}} \\ \vdots \\ e_L^{\mathrm{dl}} \end{bmatrix}.$$

  - Total transmit power $\quad P^{\mathrm{dl}}(\{p_i^{\mathrm{dl}}\}, \{q_l^{\mathrm{dl}}\}) = \sum_{i=1}^{K} p_i^{\mathrm{dl}} + \sum_{l=1}^{L} q_l^{\mathrm{dl}}$
  - Fronthaul link rate

$$C_l^{\mathrm{dl}}(\{p_i^{\mathrm{dl}}, \mathbf{v}_i\}, q_l^{\mathrm{dl}}) = \log_2 \frac{\sum_{i=1}^{K} p_i^{\mathrm{dl}} |v_{i,l}|^2 + q_l^{\mathrm{dl}}}{q_l^{\mathrm{dl}}}$$

  - End-to-end rate

$$R_k^{\mathrm{dl}}(\{p_i^{\mathrm{dl}}, \mathbf{v}_i\}, \{q_l^{\mathrm{dl}}\}) = \log_2 \frac{\sum_{i=1}^{K} p_i^{\mathrm{dl}} |\mathbf{v}_i^H \mathbf{h}_k|^2 + \sum_{l=1}^{L} q_l^{\mathrm{dl}} |h_{l,k}|^2 + \sigma^2}{\sum_{j=k} p_j^{\mathrm{dl}} |\mathbf{v}_j^H \mathbf{h}_k|^2 + \sum_{l=1}^{L} q_l^{\mathrm{dl}} |h_{l,k}|^2 + \sigma^2}$$

Uplink-Downlink Duality in C-RAN

- Any achievable rate tuple in the uplink is also achievable in the downlink

$$
\begin{aligned}
\text{find}\ & \{p_i^{\text{dl}}, \mathbf{v}_i\}, \{q_l^{\text{dl}}\} \\
\text{s.t.}\ & R_k^{\text{dl}}(\{p_i^{\text{dl}}, \mathbf{v}_i\}, \{q_l^{\text{dl}}\}) = R_k^{\text{ul}}(\{\bar{p}_i^{\text{ul}}, \bar{\mathbf{w}}_i\}, \{\bar{q}_l^{\text{ul}}\}), \quad k, \\
& C_l^{\text{dl}}(\{p_i^{\text{dl}}, \mathbf{v}_i\}, q_l^{\text{dl}}) = C_l^{\text{ul}}(\{\bar{p}_i^{\text{ul}}\}, \bar{q}_l^{\text{ul}}), \quad l, \\
& P^{\text{dl}}(\{p_i^{\text{dl}}\}, \{q_l^{\text{dl}}\}) = P^{\text{ul}}(\{\bar{p}_i^{\text{ul}}\}).
\end{aligned}
$$

Uplink-Downlink Duality in C-RAN

- Any achievable rate tuple in the downlink is also achievable in the uplink

$$
\begin{aligned}
\text{find } & \{p_i^{\mathrm{ul}}, \mathbf{w}_i\}, \{q_l^{\mathrm{ul}}\} \\
\text{s.t. } & R_k^{\mathrm{ul}}(\{p_i^{\mathrm{ul}}, \mathbf{w}_i\}, \{q_l^{\mathrm{ul}}\}) = R_k^{\mathrm{dl}}(\{\bar{p}_i^{\mathrm{dl}}, \bar{\mathbf{v}}_i\}, \{\bar{q}_l^{\mathrm{dl}}\}), \quad k, \\
& C_l^{\mathrm{ul}}(\{p_i^{\mathrm{ul}}\}, q_l^{\mathrm{ul}}) = C_l^{\mathrm{dl}}(\{\bar{p}_i^{\mathrm{dl}}, \bar{\mathbf{v}}_i\}, \bar{q}_l^{\mathrm{dl}}), \quad l, \\
& P^{\mathrm{ul}}(\{p_i^{\mathrm{ul}}\}) = P^{\mathrm{dl}}(\{\bar{p}_i^{\mathrm{dl}}\}, \{\bar{q}_l^{\mathrm{dl}}\}).
\end{aligned}
$$

- Identical achievable rate regions under the same sum-power and individual fronthaul capacity constraints

Uplink-Downlink Duality in C-RAN

- Application: sum-power minimization
- Uplink: fixed-point method

$$
\begin{aligned}
\underset{\{p_i^{\mathrm{ul}},\mathbf{w}_i\},\{q_l^{\mathrm{ul}}\}}{\text{minimize}} \quad & P^{\mathrm{ul}}(\{p_i^{\mathrm{ul}}\}) \\
\text{subject to} \quad & R_k^{\mathrm{ul}}(\{p_i^{\mathrm{ul}},\mathbf{w}_i\},\{q_l^{\mathrm{ul}}\}) \geq R_k, \qquad k, \\
& C_l^{\mathrm{ul}}(\{p_i^{\mathrm{ul}}\},q_l^{\mathrm{ul}}) \leq C_l, \qquad l.
\end{aligned}
$$

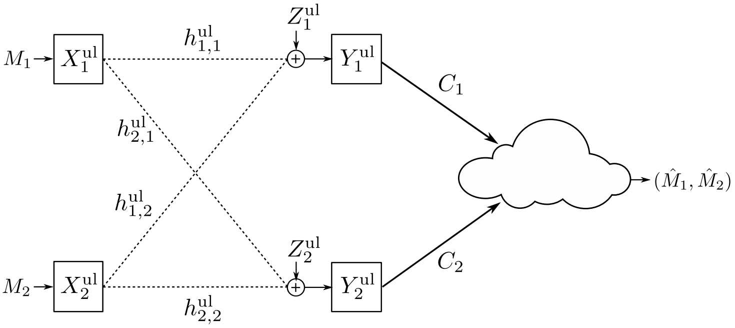
- Downlink: based on uplink solution

$$
\begin{aligned}
\underset{\{p_i^{\mathrm{dl}},\mathbf{v}_i\},\{q_l^{\mathrm{dl}}\}}{\text{minimize}} \quad & P^{\mathrm{dl}}(\{p_i^{\mathrm{dl}}\},\{q_l^{\mathrm{dl}}\}) \\
\text{subject to} \quad & R_k^{\mathrm{dl}}(\{p_i^{\mathrm{dl}},\mathbf{v}_i\},\{q_l^{\mathrm{dl}}\}) \geq R_k, \qquad k, \\
& C_l^{\mathrm{dl}}(\{p_i^{\mathrm{dl}},\mathbf{v}_i\},q_l^{\mathrm{dl}}) \leq C_l, \qquad l.
\end{aligned}
$$

Uplink-Downlink Duality in C-RAN

- Remark 1: duality holds for dirty paper coding and successive interference cancellation
- Remark 2: duality holds for per-RRH power constraint
  - Application: weighted sum-rate maximization subject to per-RRH power constraint

Compression Strategy for Downlink C-RAN



Successive decoding region for MAC

$$R_1 < I(X_1^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}} | X_2^{\mathrm{ul}});$$
$$R_2 < I(X_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}} | X_1^{\mathrm{ul}});$$
$$R_1 + R_2 < I(X_1^{\mathrm{ul}}, X_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}})$$

Wyner-Ziv Compression

$$C_1 > I(Y_1^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}} | \hat{Y}_2^{\mathrm{ul}});$$
$$C_2 > I(Y_2^{\mathrm{ul}}; \hat{Y}_2^{\mathrm{ul}} | \hat{Y}_1^{\mathrm{ul}});$$
$$C_1 + C_2 > I(Y_1^{\mathrm{ul}}, Y_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}})$$

Achievable rate-region for NNC

$$
\begin{aligned}
R_1 &< I(X_1^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}} | X_2^{\mathrm{ul}});\\
R_1 &< I(X_1^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}} | X_1^{\mathrm{ul}}) + C_2 - I(Y_2^{\mathrm{ul}}; \hat{Y}_2^{\mathrm{ul}} | X_1^{\mathrm{ul}}, X_2^{\mathrm{ul}}, \hat{Y}_1^{\mathrm{ul}});\\
R_1 &< I(X_1^{\mathrm{ul}}; \hat{Y}_2^{\mathrm{ul}} | X_2^{\mathrm{ul}}) + C_1 - I(Y_1^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}} | X_1^{\mathrm{ul}}, X_2^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}});\\
R_1 &< C_1 + C_2 - I(Y_1^{\mathrm{ul}}, Y_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}} | X_1^{\mathrm{ul}}, X_2^{\mathrm{ul}});\\
R_2 &< I(X_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}} | X_1^{\mathrm{ul}});\\
R_2 &< I(X_2^{\mathrm{ul}}; \hat{Y}_2^{\mathrm{ul}} | X_1^{\mathrm{ul}}) + C_1 - I(Y_1^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}} | X_1^{\mathrm{ul}}, X_2^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}});\\
R_2 &< I(X_1^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}} | X_2^{\mathrm{ul}}) + C_2 - I(Y_2^{\mathrm{ul}}; \hat{Y}_2^{\mathrm{ul}} | X_1^{\mathrm{ul}}, X_2^{\mathrm{ul}}, \hat{Y}_1^{\mathrm{ul}});\\
R_2 &< C_1 + C_2 - I(Y_1^{\mathrm{ul}}, Y_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}} | X_1^{\mathrm{ul}}, X_2^{\mathrm{ul}});\\
R_1 + R_2 &< I(X_1^{\mathrm{ul}}, X_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}});\\
R_1 + R_2 &< I(X_1^{\mathrm{ul}}, X_2^{\mathrm{ul}}; \hat{Y}_2^{\mathrm{ul}}) + C_1 - I(Y_1^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}} | X_1^{\mathrm{ul}}, X_2^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}});\\
R_1 + R_2 &< I(X_1^{\mathrm{ul}}, X_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}) + C_2 - I(Y_2^{\mathrm{ul}}; \hat{Y}_2^{\mathrm{ul}} | X_2^{\mathrm{ul}}, X_1^{\mathrm{ul}}, \hat{Y}_1^{\mathrm{ul}});\\
R_1 + R_2 &< C_1 + C_2 - I(Y_1^{\mathrm{ul}}, Y_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}} | X_1^{\mathrm{ul}}, X_2^{\mathrm{ul}})
\end{aligned}
$$

Simplified achievable rate-region for NNC

$$
\begin{aligned}
R_1 &< I(X_1^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}} | X_2^{\mathrm{ul}});\\
R_1 &< I(X_1^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}} | X_2^{\mathrm{ul}}) + C_1 - I(Y_1^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}} | \hat{Y}_2^{\mathrm{ul}}, X_2^{\mathrm{ul}});\\
R_1 &< I(X_1^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}} | X_2^{\mathrm{ul}}) + C_2 - I(Y_2^{\mathrm{ul}}; \hat{Y}_2^{\mathrm{ul}} | \hat{Y}_1^{\mathrm{ul}}, X_2^{\mathrm{ul}});\\
R_1 &< I(X_1^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}} | X_2^{\mathrm{ul}}) + C_1 + C_2 - I(Y_1^{\mathrm{ul}}, Y_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}} | X_2^{\mathrm{ul}});\\
R_2 &< I(X_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}} | X_1^{\mathrm{ul}});\\
R_2 &< I(X_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}} | X_1^{\mathrm{ul}}) + C_1 - I(Y_1^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}} | \hat{Y}_2^{\mathrm{ul}}, X_1^{\mathrm{ul}});\\
R_2 &< I(X_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}} | X_1^{\mathrm{ul}}) + C_2 - I(Y_2^{\mathrm{ul}}; \hat{Y}_2^{\mathrm{ul}} | \hat{Y}_1^{\mathrm{ul}}, X_1^{\mathrm{ul}});\\
R_2 &< I(X_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}} | X_1^{\mathrm{ul}}) + C_1 + C_2 - I(Y_1^{\mathrm{ul}}, Y_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}} | X_1^{\mathrm{ul}});\\
R_1 + R_2 &< I(X_1^{\mathrm{ul}}, X_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}});\\
R_1 + R_2 &< I(X_1^{\mathrm{ul}}, X_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}}) + C_1 - I(Y_1^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}} | \hat{Y}_2^{\mathrm{ul}});\\
R_1 + R_2 &< I(X_1^{\mathrm{ul}}, X_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}}) + C_2 - I(Y_2^{\mathrm{ul}}; \hat{Y}_2^{\mathrm{ul}} | \hat{Y}_1^{\mathrm{ul}});\\
R_1 + R_2 &< I(X_1^{\mathrm{ul}}, X_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}}) + C_1 + C_2 - I(Y_1^{\mathrm{ul}}, Y_2^{\mathrm{ul}}; \hat{Y}_1^{\mathrm{ul}}, \hat{Y}_2^{\mathrm{ul}})
\end{aligned}
$$

Simplified achievable rate-region for DDF

$$
\begin{aligned}
R_1 &< I(U_1, Y_1^{\mathrm{dl}}); \\
R_1 &< I(U_1, Y_1^{\mathrm{dl}}) + C_1 - I(U_1; X_1^{\mathrm{dl}}); \\
R_1 &< I(U_1, Y_1^{\mathrm{dl}}) + C_2 - I(U_1; X_2^{\mathrm{dl}}); \\
R_1 &< I(U_1, Y_1^{\mathrm{dl}}) + C_1 + C_2 - I(U_1; X_1^{\mathrm{dl}}, X_2^{\mathrm{dl}}); \\
R_2 &< I(U_2, Y_2^{\mathrm{dl}}); \\
R_2 &< I(U_2, Y_2^{\mathrm{dl}}) + C_1 - I(U_2; X_1^{\mathrm{dl}}); \\
R_2 &< I(U_2, Y_2^{\mathrm{dl}}) + C_2 - I(U_2; X_2^{\mathrm{dl}}); \\
R_2 &< I(U_2, Y_2^{\mathrm{dl}}) + C_1 + C_2 - I(U_2; X_1^{\mathrm{dl}}, X_2^{\mathrm{dl}}); \\
R_1 + R_2 &< I(U_1, Y_1^{\mathrm{dl}}) + I(U_2, Y_2^{\mathrm{dl}}) - I(U_1; U_2); \\
R_1 + R_2 &< I(U_1, Y_1^{\mathrm{dl}}) + I(U_2, Y_2^{\mathrm{dl}}) - I(U_1; U_2) + C_1 - I(U_1, U_2; X_1^{\mathrm{dl}}); \\
R_1 + R_2 &< I(U_1, Y_1^{\mathrm{dl}}) + I(U_2, Y_2^{\mathrm{dl}}) - I(U_1; U_2) + C_2 - I(U_1, U_2; X_2^{\mathrm{dl}}); \\
R_1 + R_2 &< I(U_1, Y_1^{\mathrm{dl}}) + I(U_2, Y_2^{\mathrm{dl}}) - I(U_1; U_2) + C_1 + C_2 - I(U_1, U_2; X_1^{\mathrm{dl}}, X_2^{\mathrm{dl}}) \\
&\quad - I(X_1^{\mathrm{dl}}; X_2^{\mathrm{dl}})
\end{aligned}
$$

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Thanks for listening!

Any questions/comments/thoughts?