An Uplink-Downlink Duality in Cloud Radio Access Network

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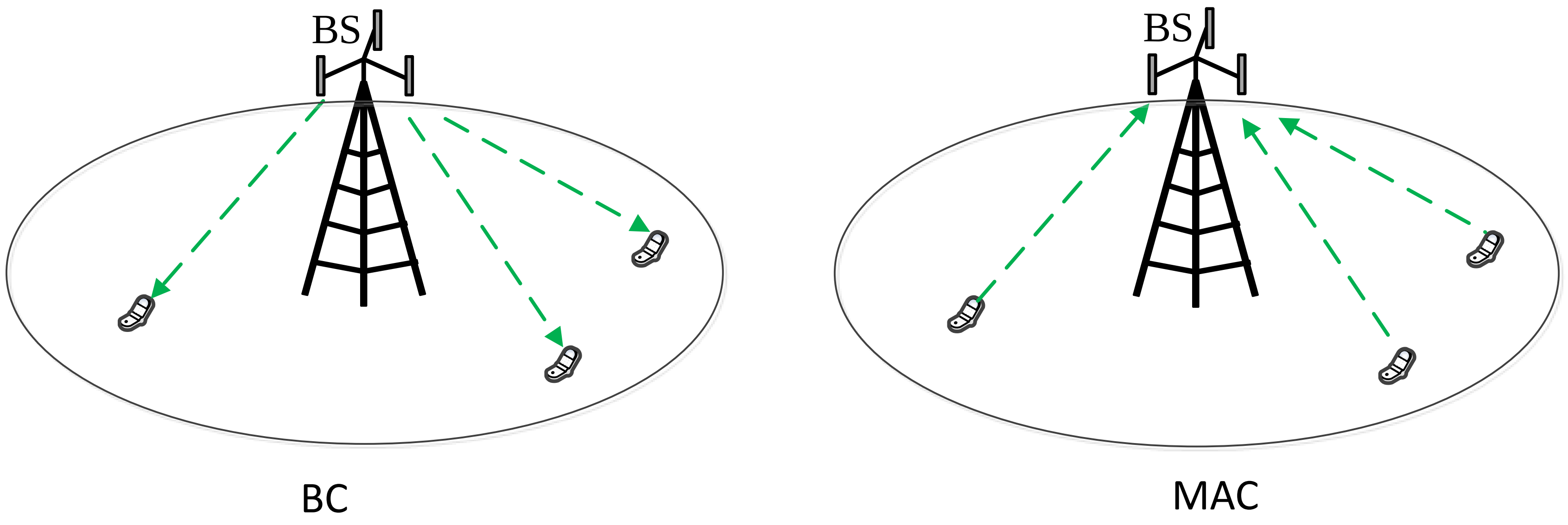
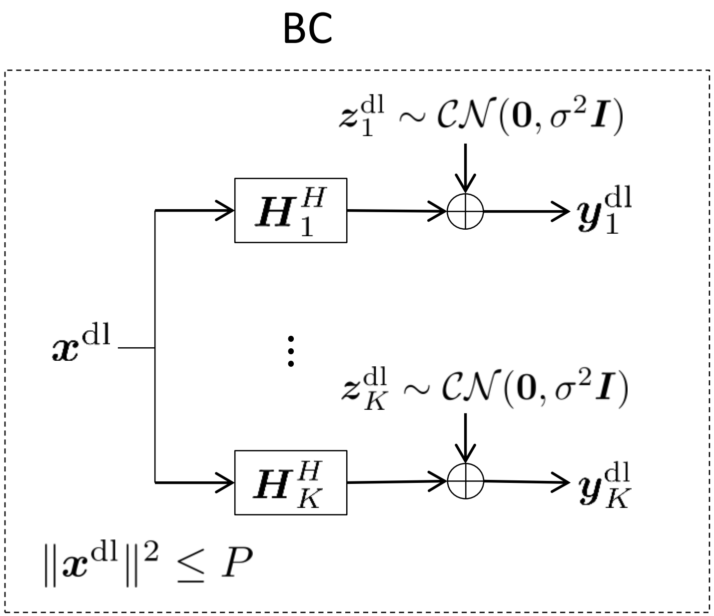
Uplink-Downlink Duality

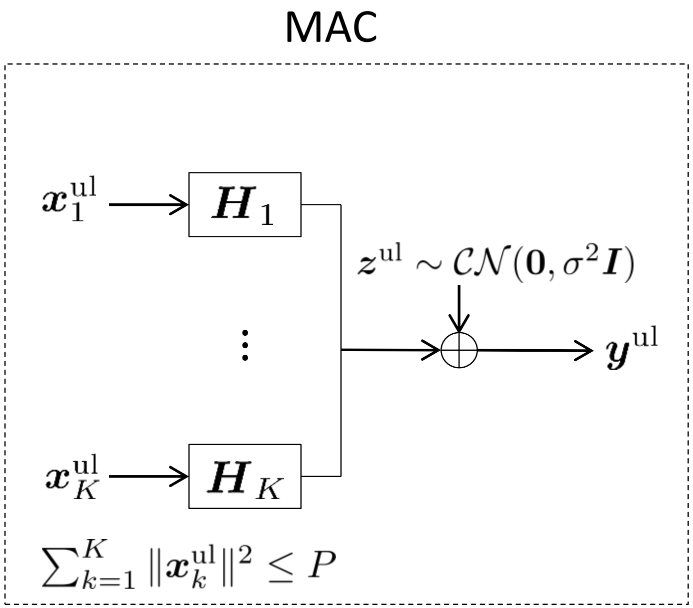
- ❑ An equivalence exists between broadcast channel (BC) and multiple-access channel (MAC)

- ❑ Capacity regions or achievable rate regions of BC and MAC are identical under certain conditions



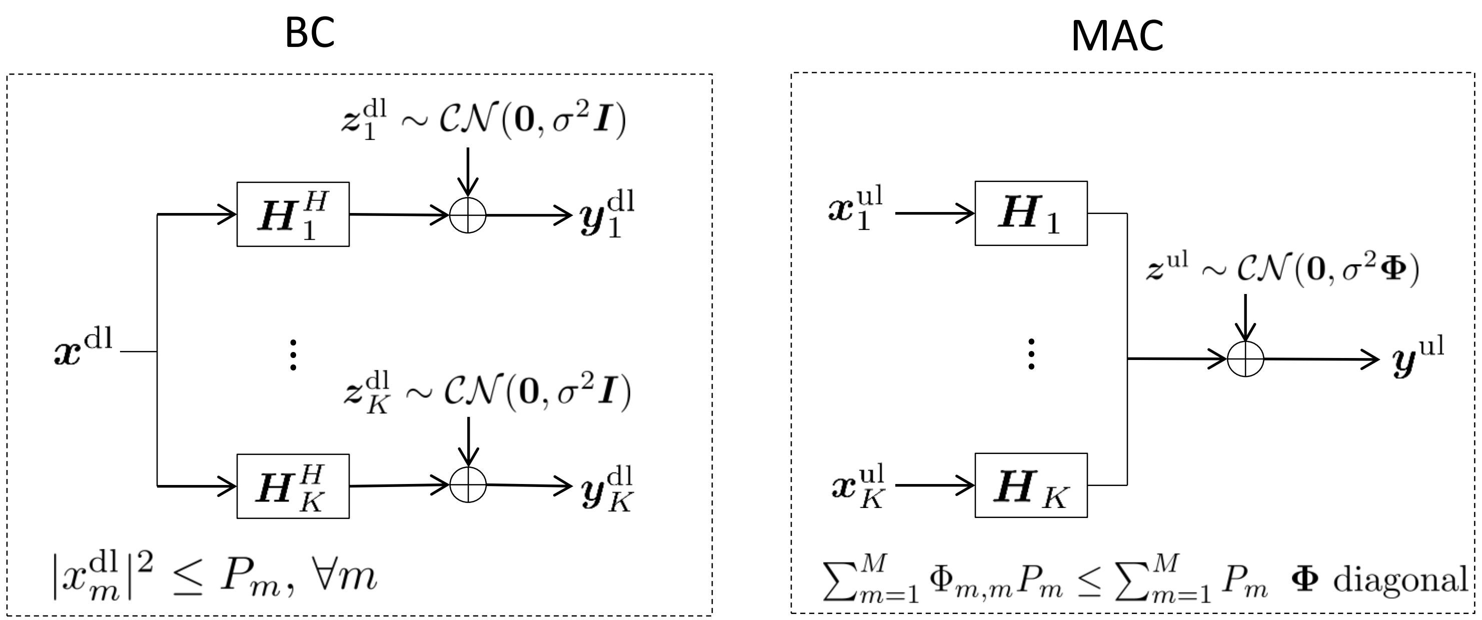
Uplink-Downlink Duality: Sum-Power Constraint





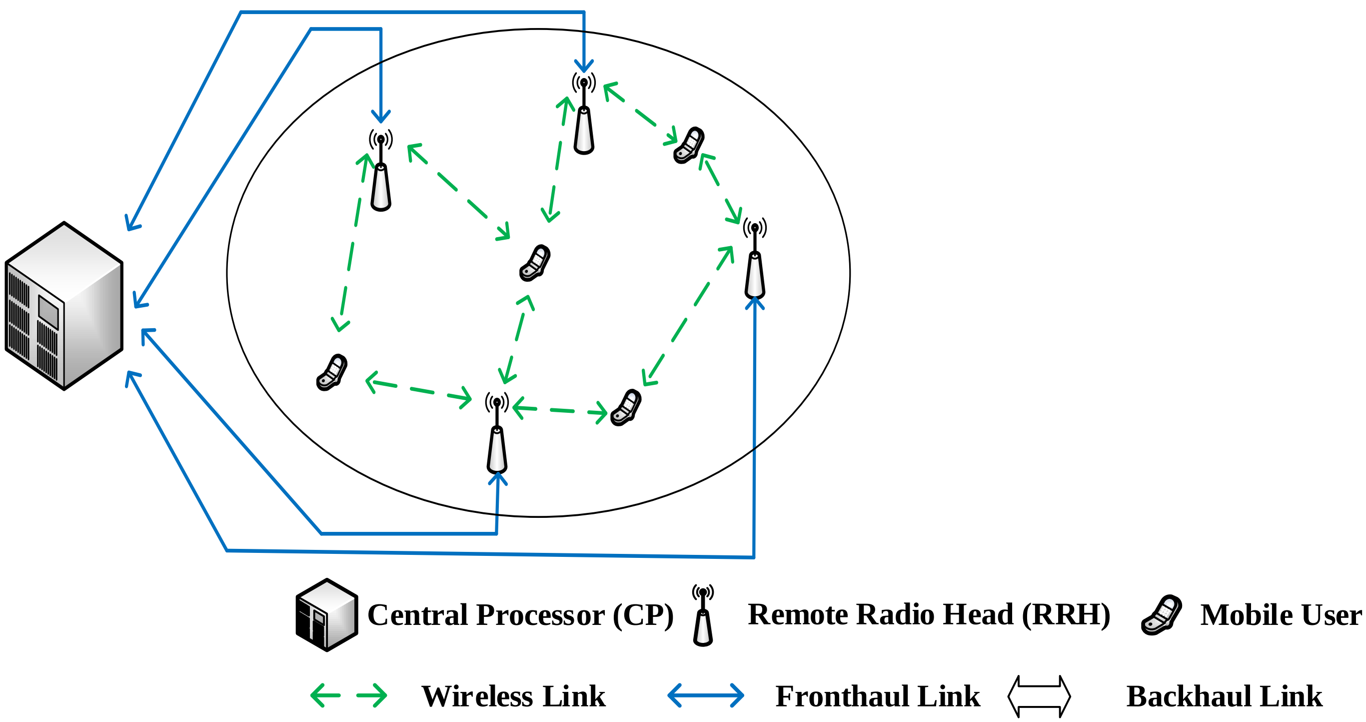
- Conditions
  - Channels: conjugate transpose
  - Noise: AWGN with covariance $\sigma^2 \boldsymbol{I}$
  - Transmit power: one sum-power constraint
- Duality
  - Linear encoding and decoding: identical achievable rate regions [Farrokhi-Liu-Tassiulas’99]
  - Dirty paper coding and successive interference cancellation: identical capacity regions [Viswanath-Tse’03] [Vishwanath-Jindal-Goldsmith’03]

Uplink-Downlink Duality: Per-Antenna Power Constraints



- ❑ Conditions
  - ➢ Channels: conjugate transpose
  - ➢ Noise: uncertain noise with covariance $\sigma^2\boldsymbol{\Phi}$
  - ➢ Transmit power: per-antenna power constraints
- ❑ Duality
  - ➢ Identical achievable rate regions or capacity regions [Yu-Lan’07]
- ❑ Can we extend existing duality result to more general channel model?

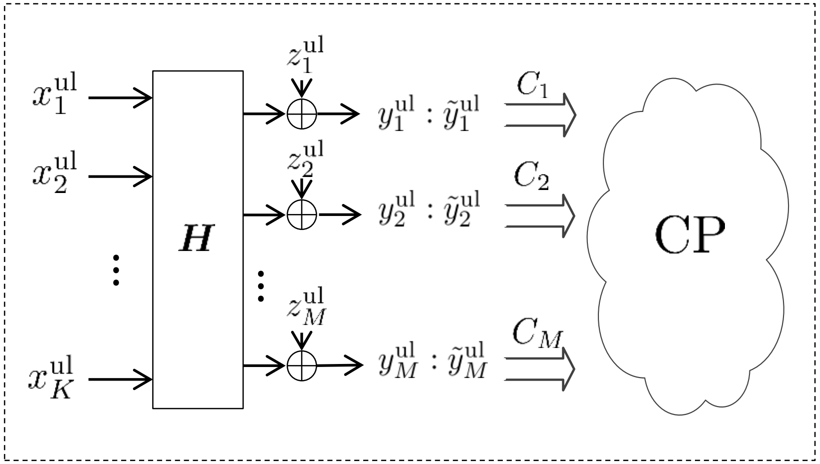
Cloud Radio Access Network



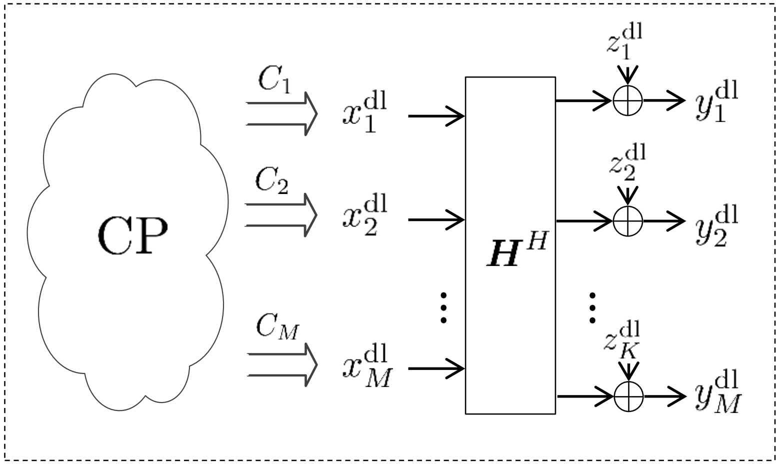
- ❑ RRHs serve users under coordination of CP via digital finite-capacity fronthaul links
  - ➢ RRH: low baseband capacility CP: joint signal processing gain
- ❑ Duality is true with infinite-capacity fronthaul links
  - ➢ Uplink: MAC Downlink: BC
- ❑ How about finite-capacity fronthaul links

Channel Model of C-RAN

Uplink C-RAN



Downlink C-RAN



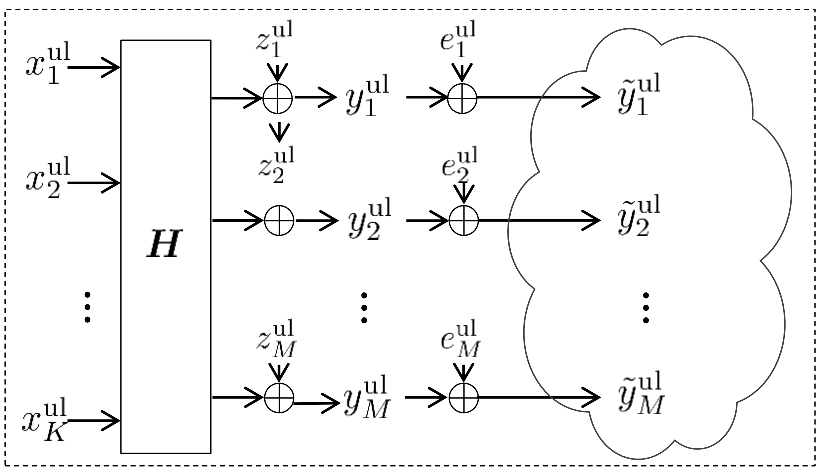
- Uplink: multiple-access relay channel
  - First hop: wireless
  - Second hop: digital
- Downlink: broadcast relay channel
  - First hop: digital
  - Second hop: wireless

- Capacity regions are in general unknown
- The same achievable rate regions between uplink and downlink C-RAN?
  - Relay strategy over the fronthaul links

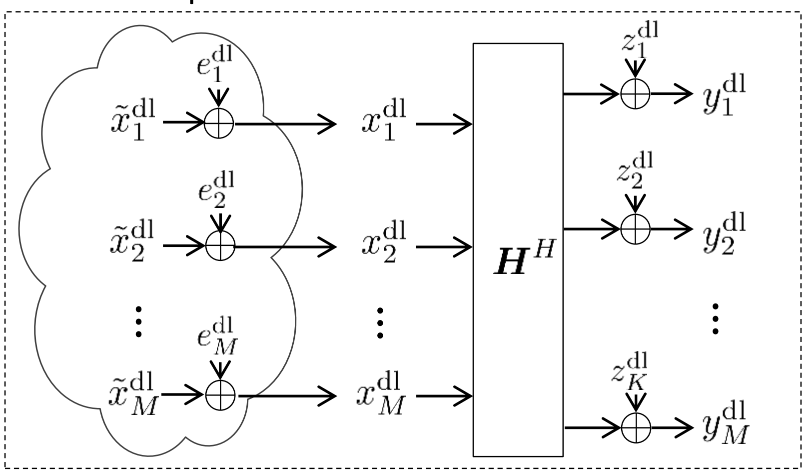
Compression-based Strategy

- Uplink: compress-and-forward [Sanderovic-Shamai-Steinberg-Kramer’08] [Zhou-Yu’14]
  - RRH compresses its received signal, then sends it to CP via fronthaul link
- Downlink: Compression-based strategy [Park-Simeone-Sahin-Shamai’13]
  - CP pre-forms beamformed signals, then compresses and transmits them to RRHs via fronthaul links

Compression-based Uplink C-RAN



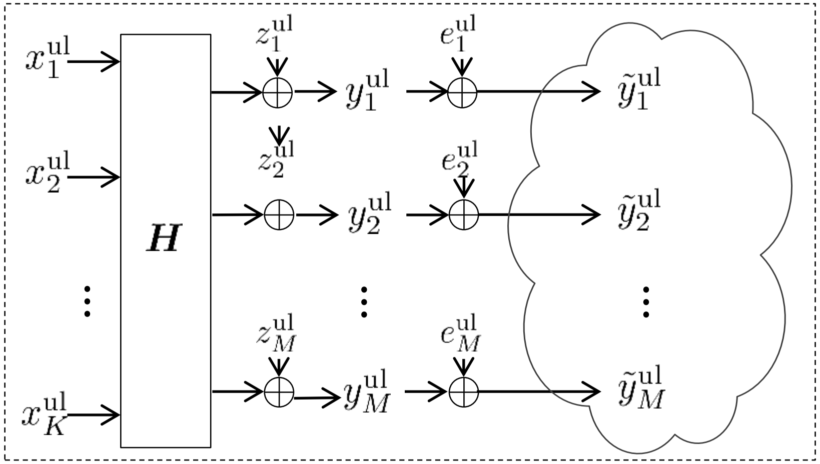
Compression-based Downlink C-RAN



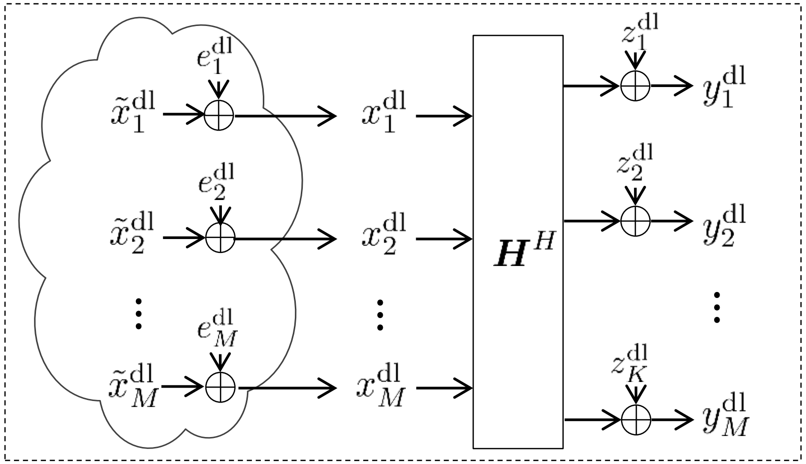
Main Result

- ❑ Main result: The duality result is true for SISO C-RAN with compression-based strategies!
  - Channels: conjugate transpose
  - Noise: AWGN with power σ^2
  - Transmit power: one sum-power constraint (later extension to per-RRH (per-antenna) power constraints)
  - Compression: independent compression across RRHs
  - Fronthaul: individual fronthaul constraints

Compression-based Uplink C-RAN



Compression-based Downlink C-RAN

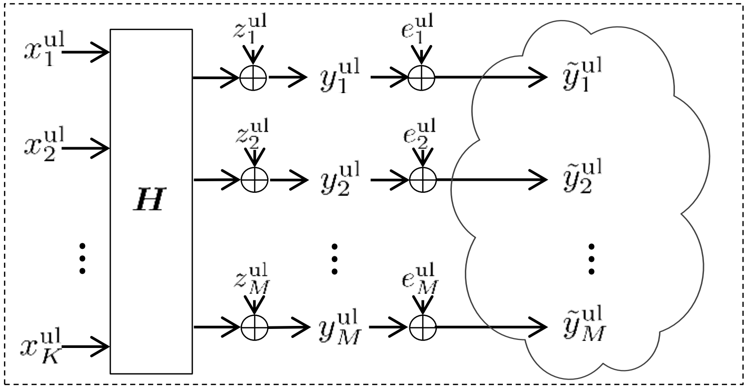


Channel Model of Uplink C-RAN

- Received signals across all RRHs

$$\begin{bmatrix} y_1^{\mathrm{ul}} \\ \vdots \\ y_M^{\mathrm{ul}} \end{bmatrix} = \begin{bmatrix} h_{1,1} & \cdots & h_{1,K} \\ \vdots & \ddots & \vdots \\ h_{M,1} & \cdots & h_{M,K} \end{bmatrix} \begin{bmatrix} x_1^{\mathrm{ul}} \\ \vdots \\ x_K^{\mathrm{ul}} \end{bmatrix} + \begin{bmatrix} z_1^{\mathrm{ul}} \\ \vdots \\ z_M^{\mathrm{ul}} \end{bmatrix}$$

with $x_k^{\mathrm{ul}} = \sqrt{p_k^{\mathrm{ul}}} s_k^{\mathrm{ul}}$



- Collected signal at CP

$$\begin{bmatrix} \tilde{y}_1^{\mathrm{ul}} \\ \vdots \\ \tilde{y}_M^{\mathrm{ul}} \end{bmatrix} = \begin{bmatrix} y_1^{\mathrm{ul}} \\ \vdots \\ y_M^{\mathrm{ul}} \end{bmatrix} + \begin{bmatrix} e_1^{\mathrm{ul}} \\ \vdots \\ e_M^{\mathrm{ul}} \end{bmatrix} \xrightarrow{\text{independent compression}} E[\boldsymbol{e}^{\mathrm{ul}}(\boldsymbol{e}^{\mathrm{ul}})^H] = \mathrm{diag}(q_1^{\mathrm{ul}}, \cdots, q_M^{\mathrm{ul}})$$

- Receive beamforming for information decoding

$$\tilde{s}_k^{\mathrm{ul}} = \boldsymbol{w}_k \tilde{\boldsymbol{y}}^{\mathrm{ul}}$$

- Designed variables: $\{p_k^{\mathrm{ul}}, \boldsymbol{w}_k\}, \{q_m^{\mathrm{ul}}\}$

Achievable Rate Region in Uplink C-RAN

- Transmit power for all users

$$P^{\text{ul}}(\{p_i^{\text{ul}}\}) = \sum_{i=1}^{K} p_i^{\text{ul}}$$

- Fronthaul rate

$$C_m^{\text{ul}}(\{p_i^{\text{ul}}\}, q_m^{\text{ul}}) = I(y_m^{\text{ul}}; \tilde{y}_m^{\text{ul}}) = \log_2 \frac{\sum_{i=1}^{K} p_i^{\text{ul}} |h_{m,i}|^2 + q_m^{\text{ul}} + \boxed{\sigma^2}}{q_m^{\text{ul}}}$$

- User rate

$$R_k^{\text{ul}}(\{p_i^{\text{ul}}, \boldsymbol{w}_i\}, \{q_m^{\text{ul}}\}) = I(s_k^{\text{ul}} : \tilde{s}_k^{\text{ul}}) = \log_2 \frac{\sum_{i=1}^{K} p_i^{\text{ul}} |\boldsymbol{w}_k^H \boldsymbol{h}_i|^2 + \sum_{m=1}^{M} q_m^{\text{ul}} |w_{k,m}|^2 + \sigma^2}{\sum_{j \neq k} p_j^{\text{ul}} |\boldsymbol{w}_k^H \boldsymbol{h}_j|^2 + \sum_{m=1}^{M} q_m^{\text{ul}} |w_{k,m}|^2 + \sigma^2}$$

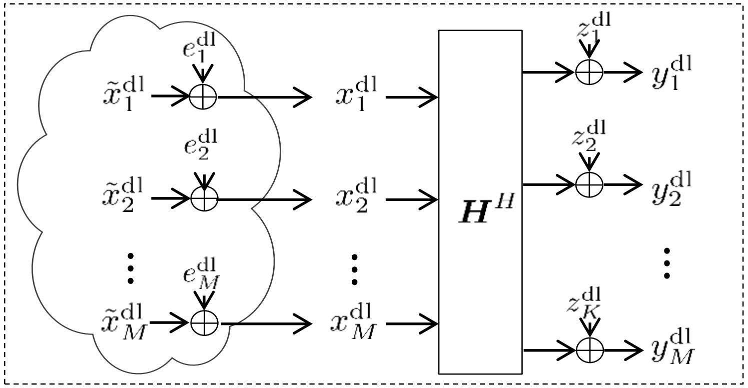
- Feasible solution set with sum-power constraint and individual fronthaul constraint

$$\begin{aligned}\mathcal{T}^{\text{ul}}(\{C_m\}, P) = \{ (\{p_i^{\text{ul}}, \boldsymbol{w}_i\}, \{q_m^{\text{ul}}\}) : P^{\text{ul}}(\{p_i^{\text{ul}}\}) \leq P, \\ C_m^{\text{ul}}(\{p_i^{\text{ul}}\}, q_m^{\text{ul}}) \leq C_m, \forall m, \|\boldsymbol{w}_i\|^2 = 1, \forall i \}\end{aligned}$$

- Achievable rate region

$$\mathcal{R}^{\text{ul}}(\{C_m\}, P) = \bigcup_{(\{p_i^{\text{ul}}, \boldsymbol{w}_i\}, \{q_m^{\text{ul}}\}) \in \mathcal{T}^{\text{ul}}(\{C_m\}, P)} \{ (r_1^{\text{ul}}, \cdots, r_K^{\text{ul}}) : r_k^{\text{ul}} \leq R_k^{\text{ul}}(\{p_i^{\text{ul}}, \boldsymbol{w}_i\}, \{q_m^{\text{ul}}\}), \forall k \}$$

Channel Model of Downlink C-RAN



- Pre-beamformed signals at CP

$$\begin{bmatrix} \tilde{x}_1^{\rm dl} \\ \vdots \\ \tilde{x}_M^{\rm dl} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^K v_{i,1}\sqrt{p_i^{\rm dl}} s_i^{\rm dl} \\ \vdots \\ \sum_{i=1}^K v_{i,M}\sqrt{p_i^{\rm dl}} s_i^{\rm dl} \end{bmatrix} = \sum_{i=1}^K \boldsymbol{v}_i \sqrt{p_i^{\rm dl}} s_i^{\rm dl}$$

- Transmit signals across RRHs

$$\begin{bmatrix} x_1^{\rm dl} \\ \vdots \\ x_M^{\rm dl} \end{bmatrix} = \begin{bmatrix} \tilde{x}_1^{\rm dl} \\ \vdots \\ \tilde{x}_M^{\rm dl} \end{bmatrix} + \begin{bmatrix} e_1^{\rm dl} \\ \vdots \\ e_M^{\rm dl} \end{bmatrix} \xrightarrow{\text{independent compression}} E[\boldsymbol{e}^{\rm dl}(\boldsymbol{e}^{\rm dl})^H] = {\rm diag}(q_1^{\rm dl}, \cdots, q_M^{\rm dl})$$

- Received signal across users

$$\begin{bmatrix} y_1^{\rm dl} \\ \vdots \\ y_K^{\rm dl} \end{bmatrix} = \begin{bmatrix} h_{1,1}^H & \cdots & h_{M,1}^H \\ \vdots & \ddots & \vdots \\ h_{1,K}^H & \cdots & h_{M,K}^H \end{bmatrix} \begin{bmatrix} x_1^{\rm dl} \\ \vdots \\ x_M^{\rm dl} \end{bmatrix} + \begin{bmatrix} z_1^{\rm dl} \\ \vdots \\ z_K^{\rm dl} \end{bmatrix}$$

- Designed variables: $\{p_k^{\rm dl}, \boldsymbol{v}_k\}, \{q_m^{\rm dl}\}$

Achievable Rate Region in Downlink C-RAN

- Transmit power for all RRHs

$$P^{\mathrm{dl}}(\{p_i^{\mathrm{dl}}\},\{q_m^{\mathrm{dl}}\}) = \sum_{i=1}^{K} p_i^{\mathrm{dl}} + \boxed{\sum_{m=1}^{M} q_m^{\mathrm{dl}}}$$

- Fronthaul rate

$$C_m^{\mathrm{dl}}(\{p_i^{\mathrm{dl}},\boldsymbol{v}_i\},q_m^{\mathrm{dl}}) = I(x_m^{\mathrm{dl}};\tilde{x}_m^{\mathrm{dl}}) = \log_2 \frac{\sum_{i=1}^{K} p_i^{\mathrm{dl}}|v_{i,m}|^2 + q_m^{\mathrm{dl}}}{q_m^{\mathrm{dl}}}$$

- User rate

$$R_k^{\mathrm{dl}}(\{p_i^{\mathrm{dl}},\boldsymbol{v}_i\},\{q_m^{\mathrm{dl}}\}) = I(s_k^{\mathrm{dl}};y_k^{\mathrm{dl}}) = \log_2 \frac{\sum_{i=1}^{K} p_i^{\mathrm{dl}}|\boldsymbol{v}_i^H\boldsymbol{h}_k|^2 + \sum_{m=1}^{M} q_m^{\mathrm{dl}}|h_{m,k}|^2 + \sigma^2}{\sum_{j\neq k} p_j^{\mathrm{dl}}|\boldsymbol{v}_j^H\boldsymbol{h}_k|^2 + \sum_{m=1}^{M} q_m^{\mathrm{dl}}|h_{m,k}|^2 + \sigma^2}$$

- Feasible solution set with sum-power constraint and individual fronthaul constraint

$$\mathcal{T}^{\mathrm{dl}}(\{C_m\},P) = \{(\{p_i^{\mathrm{dl}},\boldsymbol{v}_i\},\{q_m^{\mathrm{dl}}\}) : P^{\mathrm{dl}}(\{p_i^{\mathrm{dl}}\},\{q_m^{\mathrm{dl}}\}) \leq P,$$
$$C_m^{\mathrm{dl}}(\{p_i^{\mathrm{dl}},\boldsymbol{v}_i\},q_m^{\mathrm{dl}}) \leq C_m, \forall m, \|\boldsymbol{v}_i\|^2 = 1, \forall i\}.$$

- Achievable rate region

$$\mathcal{R}^{\mathrm{dl}}(\{C_m\},P) = \bigcup_{(\{p_i^{\mathrm{dl}},\boldsymbol{v}_i\},\{q_m^{\mathrm{dl}}\})\in\mathcal{T}^{\mathrm{dl}}(\{C_m\},P)} \{(r_1^{\mathrm{dl}},\cdots,r_K^{\mathrm{dl}}) : r_k^{\mathrm{dl}} \leq R_k^{\mathrm{dl}}(\{p_i^{\mathrm{dl}},\boldsymbol{v}_i\},\{q_m^{\mathrm{dl}}\}), \forall k\}$$

Proof of Uplink-Downlink Duality

- Any achievable rate tuple in the uplink is also achievable in the downlink

$$
\begin{aligned}
\text{find } & \{p_i^{\mathrm{dl}}, \boldsymbol{v}_i\}, \{q_m^{\mathrm{dl}}\} \\
\text{s.t. } & R_k^{\mathrm{dl}}(\{p_i^{\mathrm{dl}}, \boldsymbol{v}_i\}, \{q_m^{\mathrm{dl}}\}) = R_k^{\mathrm{ul}}(\{\bar{p}_i^{\mathrm{ul}}, \bar{\boldsymbol{w}}_i\}, \{\bar{q}_m^{\mathrm{ul}}\}), \ \forall k, \\
& C_m^{\mathrm{dl}}(\{p_i^{\mathrm{dl}}, \boldsymbol{v}_i\}, q_m^{\mathrm{dl}}) = C_m^{\mathrm{ul}}(\{\bar{p}_i^{\mathrm{ul}}\}, \bar{q}_m^{\mathrm{ul}}), \ \forall m, \\
& P^{\mathrm{dl}}(\{p_i^{\mathrm{dl}}\}, \{q_m^{\mathrm{dl}}\}) = P^{\mathrm{ul}}(\{\bar{p}_i^{\mathrm{ul}}\}).
\end{aligned}
$$

- Any achievable rate tuple in the downlink is also achievable in the uplink

$$
\begin{aligned}
\text{find } & \{p_i^{\mathrm{ul}}, \boldsymbol{w}_i\}, \{q_m^{\mathrm{ul}}\} \\
\text{s.t. } & R_k^{\mathrm{ul}}(\{p_i^{\mathrm{ul}}, \boldsymbol{w}_i\}, \{q_m^{\mathrm{ul}}\}) = R_k^{\mathrm{dl}}(\{\bar{p}_i^{\mathrm{dl}}, \bar{\boldsymbol{v}}_i\}, \{\bar{q}_m^{\mathrm{dl}}\}), \forall k, \\
& C_m^{\mathrm{ul}}(\{p_i^{\mathrm{ul}}\}, q_m^{\mathrm{ul}}) = C_m^{\mathrm{dl}}(\{\bar{p}_i^{\mathrm{dl}}, \bar{\boldsymbol{v}}_i\}, \bar{q}_m^{\mathrm{dl}}), \ \forall m, \\
& P^{\mathrm{ul}}(\{p_i^{\mathrm{ul}}\}) = P^{\mathrm{dl}}(\{\bar{p}_i^{\mathrm{dl}}\}, \{\bar{q}_m^{\mathrm{dl}}\}).
\end{aligned}
$$

- Identical achievable rate regions under the same sum-power and individual fronthaul capacity constraints

Further Remarks

- Application: sum-power minimization

Uplink

$$\begin{aligned} \underset{\{p_i^{\text{ul}},\boldsymbol{w}_i\},\{q_m^{\text{ul}}\}}{\text{minimize}} \quad & P^{\text{ul}}(\{p_i^{\text{ul}}\}) \\ \text{subject to} \quad & R_k^{\text{ul}}(\{p_i^{\text{ul}},\boldsymbol{w}_i\},\{q_m^{\text{ul}}\}) \geq R_k, \ \forall k, \\ & C_m^{\text{ul}}(\{p_i^{\text{ul}}\},q_m^{\text{ul}}) \leq C_m, \ \forall m. \end{aligned}$$

$\Rightarrow$

Downlink

$$\begin{aligned} \underset{\{p_i^{\text{dl}},\boldsymbol{v}_i\},\{q_m^{\text{dl}}\}}{\text{minimize}} \quad & P^{\text{dl}}(\{p_i^{\text{dl}}\},\{q_m^{\text{dl}}\}) \\ \text{subject to} \quad & R_k^{\text{dl}}(\{p_i^{\text{dl}},\boldsymbol{v}_i\},\{q_m^{\text{dl}}\}) \geq R_k, \ \forall k, \\ & C_m^{\text{dl}}(\{p_i^{\text{dl}},\boldsymbol{v}_i\},q_m^{\text{dl}}) \leq C_m, \ \forall m. \end{aligned}$$

- Optimal receive beamforming is MMSE
- Optimally solvable based on fixed-point method

- Optimal transmit beamforming is difficult to obtain
- Optimally solvable based on dual solution in uplink

- Remark 1: applicable to dirty paper coding and successive interference cancellation

- Remark 2: applicable to per-RRH power constraints
  - Application: downlink weighted sum-rate maximization with per-RRH power constraints

Conclusions

- A more general uplink-downlink duality in C-RAN
  - MAC ⟺ BC ⟹ multiple-access relay channel ⟺ broadcast relay channel
- Downlink C-RAN solvable based on dual uplink counterpart
  - Power minimization
  - Weighted sum-rate maximization
- Open problems
  - Multiple multi-antenna RRHs and multiple multi-antenna users
  - Wyner-Ziv compression and multivariate compression
  - Duality for other relay strategies?
    - Compute-and-forward [Hong-Caire'13]
    - Decode-and-forward: data-sharing in downlink [Dai-Yu'14]; uplink?

Thank you!